The Diffusion Model of Turbulence

Implications from dissipation due to reconnection

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Magnetic Reconnection and Plasma Turbulence
Uppsala 29 May 2007

Outline

1. Motivation
   - Reconnection as a channel for dissipation in turbulent plasma
   - The problem with the dissipation range
2. Turbulence as diffusive energy transfer
   - Diffusion-advection phenomenology
   - The effect of reconnection
3. Results
   - Numerical Implementation
   - Input Sources and Sinks
   - Main Results

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Observation of Reconnection in Turbulence

- Reconnection in thin current sheets in turbulence.
- Energy conversion of magnetic energy to particle kinetic and thermal energies ($\mathbf{j} \cdot \mathbf{E}$ measured).
- Abundance of thin current sheets $\Rightarrow$ efficient dissipation mechanism.
- Thickness of current sheets around $\lambda_i = c/\omega_i$.
- Dissipation rates $D_{reconnection}/D_{damping} \sim 100$ at length scales λ_i.

[Retinò et al., 2007, Sundkvist et al., 2007]

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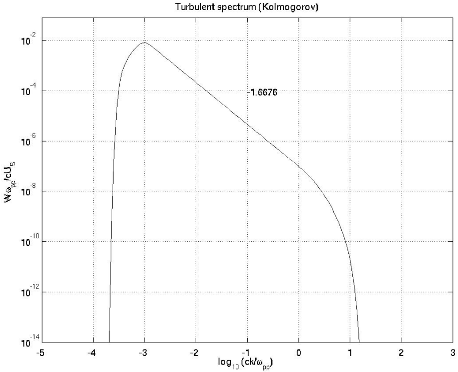
Theory (vs Observations)



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- Wave damping → Exponential cut-off in dissipation range [Li et al., 2001]

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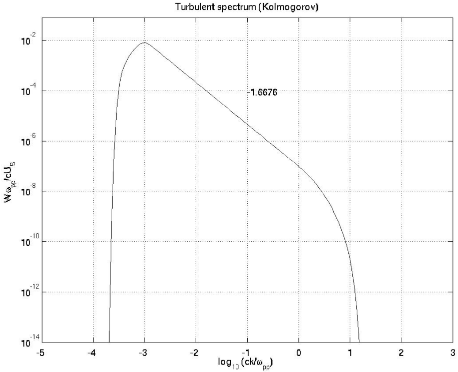

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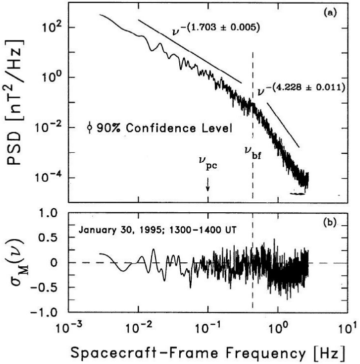

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- Observations in Solar wind [Leamon et al., 1998]
- Second “power law” observed, not dissipation (exp cut-off)

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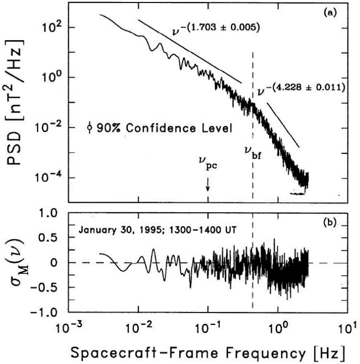

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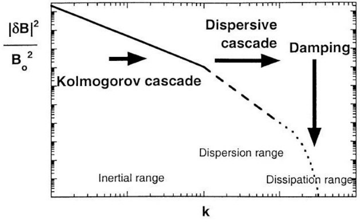

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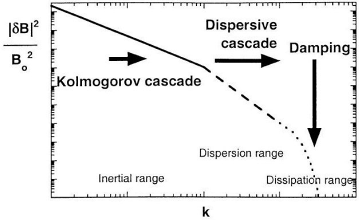

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Purpose of study

- Use simple model (isotropic, Kolmogorov) of MHD turbulence to investigate the observable effects of dissipation due to reconnection.

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Turbulence as Energy Diffusion in Wavenumber Space

- Developed for hydrodynamical turbulence by [Leith, 1967]
- Generalized to MHD turbulence [Zhou and Matthaeus, 1990].
- Applied in a varitey of contexts.

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Phenomenology of Diffusive Turbulent Energy Transfer

- Transport equation for 3-dim spectral density $\widehat{W}(\mathbf{k})$

$$\frac{\partial \widehat{W}(\mathbf{k})}{\partial t} = -\nabla_{\mathbf{k}} \cdot \mathbf{F}(\mathbf{k})$$

  where the flux of energy in wavenumber space is

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Isotropic Case

- One-dimensional energy spectrum $W(k) = 4\pi k^2 \widehat{W}(\mathbf{k})$ gives

$$\frac{\partial W(k)}{\partial t} = \underbrace{\frac{\partial}{\partial k}\left[k^2 D(k)\frac{\partial}{\partial k}\left(\frac{k^{-2}}{4\pi}W(k)\right)\right]}_{\text{diffusive transfer in inertial range}} + \underbrace{S(k) - \gamma(k)W(k)}_{\text{Sources and sinks}}$$

- Kolmogorov wavenumber diffusion coefficient

$$D(k) = C^2 v_A k^{7/2}\left(\frac{W(k)}{2U_B}\right)^{1/2}$$

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The Flux Term

- Can be written as

$$\frac{\partial}{\partial \tilde{k}}\left[\tilde{k}^{11/2}\tilde{W}(\tilde{k})^{1/2}\frac{\partial}{\partial \tilde{k}}\left(\tilde{k}^{-2}\tilde{W}(\tilde{k})\right)\right] = \frac{\partial F(\tilde{k},t)}{\partial \tilde{k}}$$

$$F(\tilde{k},t) = D(\tilde{k},t)\frac{\partial \tilde{W}}{\partial \tilde{k}} + A(\tilde{k},t)\tilde{W}$$

with the diffusion and advection coefficients being

$$D(\tilde{k},t) = \tilde{k}^{7/2}$$

$$A(\tilde{k},t) = -2\tilde{k}^{5/2}\tilde{W}^{1/2}$$

Sources and sinks

$$\frac{\partial W(k)}{\partial t} = \frac{\partial}{\partial k}\left[k^2 D(k) \frac{\partial}{\partial k}\left(\frac{k^{-2}}{4\pi} W(k)\right)\right] + S(k) - \gamma(k) W(k)$$

- Source injection at $k = k_0$:

$$S(k) = S(k_0)\delta(k - k_0)$$

- Dissipation by damping of (quasi-)linear waves

$$\gamma(k) = \gamma_0 k^2 H(k - k_d)$$

k^2 dependence consistent with magnetosonic waves

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Observational Constraints and Model

- Localized to length scales around λ_i.
- Dissipation rates $D_{reconnection}/D_{damping} \sim 100$ around λ_i.

New term capturing observational features $\Rightarrow$

$$\frac{\partial W(k)}{\partial t} = \frac{\partial}{\partial k}\left[k^2 D(k)\frac{\partial}{\partial k}\left(\frac{k^{-2}}{4\pi}W(k)\right)\right]+S(k)-\gamma(k)W(k)-R(k)$$

- Model dissipation due to reconnection with

$$R(k)=\gamma_R(k)W(k)=R_0\frac{1}{\sqrt{\pi\epsilon_R}}e^{-(k-k_R)^2/\epsilon_R}W(k)$$

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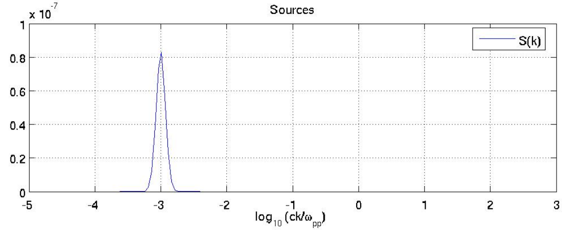
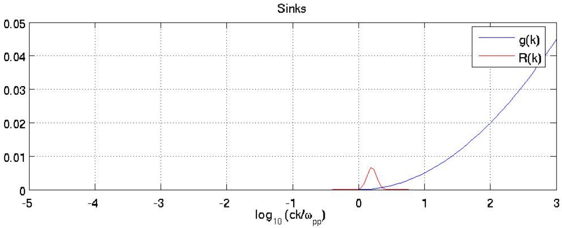
Example of Sources and Sinks





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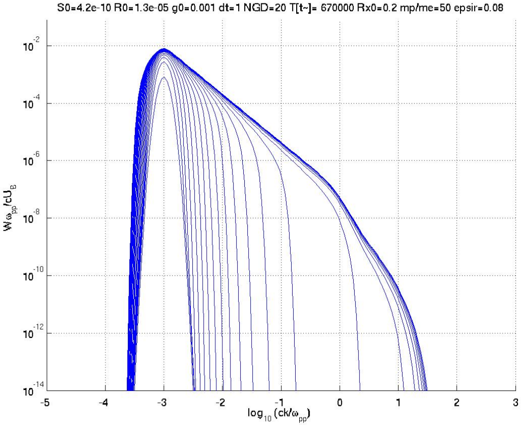
Temporal Evolution of Solution



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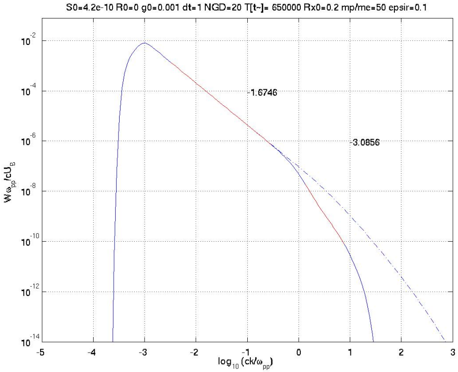
Altered Behaviour for Small Scales



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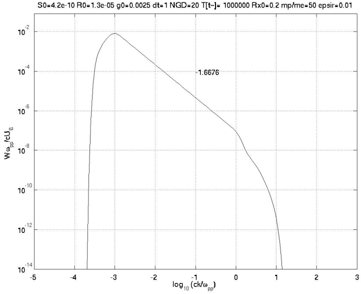
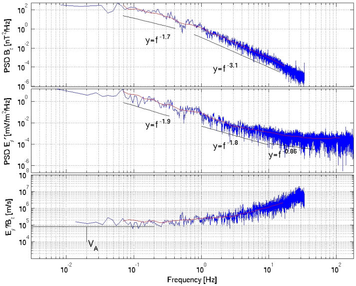
- Effect of dissipation due to reconnection can be “observational power law”

Altered Behaviour for Small Scales II





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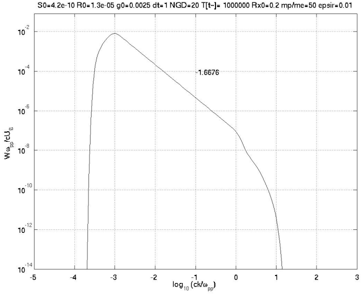

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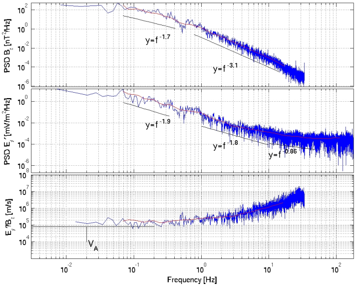

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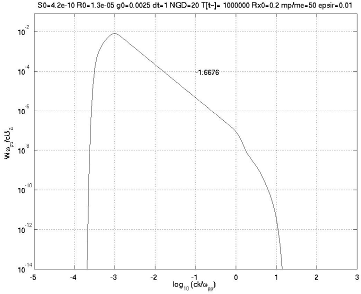

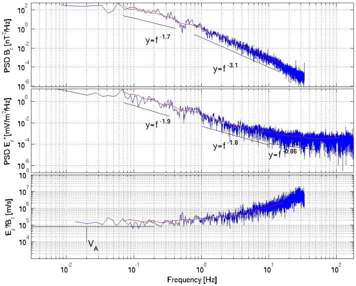

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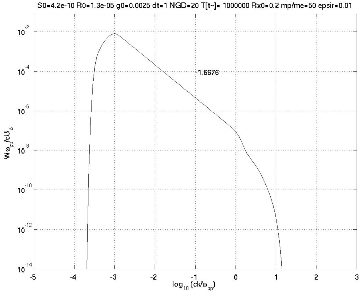

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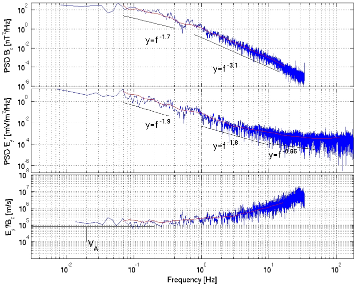

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Summary

- Considered a diffusive energy transport model for turbulence.
- Modeled the effect of dissipation due to reconnection localized around λ_i with new term.
- Effect of reconnection and damping together $\Rightarrow$ "observational power law"
- Narrow region of reconnection scale sizes creates dip in spectra.
- Outlook
  - Additional effects from dispersion (accelerated diffusion for dispersion range wavenumbers) to be included.

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For Further Reading I

Leamon, R. J., Smith, C. W., Ness, N. F., Matthaeus, W. H., and Wong, H. K. (1998).
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Models of inertial range spectra of interplanetary magnetohydrodynamic turbulence.
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