Plasma pressure relief and generation of field-aligned currents in the magnetosphere.

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CALCULATION OF PLASMA PRESSURE DISTRIBUTION IN THE MAGNETOSPHERE

The equations of the two-fluid or one-fluid magnetohydrodynamics with isotropic or anisotropic pressure are as a rule used to describe magnetospheric plasma. In this case any dissipative processes in the system are considered inessential. This statement is usually valid for ohmic loss and loss by radiation. However, particles (and energy) also escape from the magnetospheric plasma into the atmosphere through open ends of flux tubes. This type of loss can be very substantial and should be taken into account. The first consequences of such loss were studied by Kennel [1969, Rev. Geophys]. We present the set of equations describing the magnetospheric plasma [Ponomarev, 1985; Ponomarev, Sedykh, 2006]:

\[
\begin{align*}
\frac{dn_i}{dt} + n_i \nabla \cdot V_i &= - \frac{n_i}{\tau_i} \\
\frac{dp_i}{dt} + \gamma p_i \nabla \cdot V_i &= - \frac{p_i}{\tau_i} \\
\nabla p &= \left[ j \times B \right]/c \\
\int dt/\tau &= \int dR/V_R \tau = \int R d\lambda/V_\lambda \tau
\end{align*}
\]

Here \(i = e, p\) is the index of electrons or protons; \(n_i\) is the particle number density; \(p_i\) is the pressure of given particles; \(t\) is the current time; \(U\) is the plasma tube volume (by plasma tube we mean the plasma content of a magnetic flux tube); \(E\) is the electric field strength; \(B\) is the magnetic field strength; \(V_i\) is the electric drift velocity of the corresponding plasma component; \(V = V_e + V_p\) is the drift velocity of plasma as a whole; and \(j\) is the electric current density. If \(V\), a difference in the drift velocities between the electron and proton components can be neglected. Hereafter, we will do so if no special assumptions are made. The condition of quasineutrality, i.e., \((n_p - n_e)/(n_p + n_e) \ll 1\), is satisfied throughout the magnetospheric plasma. The first equation in the set is the continuity equation for electrons and ions taking into account particle loss due to pitch-angle diffusion into the loss cone. The characteristic time is the time over which the plasma tube loses \(1/e\) part of the initial number of particles. Equations (2) describe the electron and proton gas pressure behavior.
during motion due to precipitation. The gas behavior is evidently nonadiabatic. Equations (3) are factually the equations of plasma motion in a quasistationary case. The first equation in (4) is the expression for entropy density. Here \( c_v \) is heat capacity at a constant volume. Entropy evidently increases in the course of time, and the process of plasma convection is irreversible in our approximation. The second expressions in (1) and (2) are the solutions to the corresponding equations. Our set of magnetohydrodynamic equations was substantiated in detail in [Ponomarev, 1985; Ponomarev, Sedykh, 2006, Geomagnetism & Aeronomy]. The applicability of this set was analyzed, and the definitions of \( \tau \) were given in the same work.

From the balance equation of gas kinetic energy in a steady-state one-dimensional case we can obtain:

\[
\begin{align*}
V \frac{dP}{dx} + \gamma \cdot p \frac{dV}{dx} &= -\gamma \frac{p}{\tau} \\
V \frac{dP}{dx} &= -p \left( \frac{dV}{dx} + \frac{\gamma}{\tau} \right) \\
\int \frac{dp}{p} &= \int \gamma \left( -\frac{dV}{V dx} \right) dx - \int \frac{\gamma}{V \tau} dx \\
\ln p &= \gamma (\ln V_0 - \ln V) + \ln p^0 - \gamma \int \frac{dx}{V \tau} \\
\int dt &= \frac{dR}{V_R} = R_0 d\lambda/V_\lambda, \quad \Delta t = \int dt \text{ is the transport time, i.e., the time over which the flux tube will move from the boundary to the given point on the flux line; and } V_R \text{ and } V_\lambda \text{ are the radial and azimuthal components of convection velocity. Thus (*) indicates how gas pressure changes when plasma moves along the}
\end{align*}
\]
convection line at a velocity $V = (V_R^2 + V_\lambda^2)^{1/2}$. Specifying the initial pressure $p_0$ at the boundary $L = L_\infty$, we can find the resultant pressure at any point on the flux line. In such a way, the field of pressures in the entire magnetosphere is calculated (Fig. 1, Fig.2).

Similarly, it is possible to obtain the following expression:

$$\frac{\gamma}{\gamma - 1}\left\{ \frac{L_\infty}{L} \right\}^{4\gamma} \left[ \exp\left( -\frac{\gamma}{\gamma - 1}\frac{dr}{V_r \tau_1} \right) + \exp\left( -\frac{\gamma}{\gamma - 1}\frac{dr}{V_g \tau_2} \right) \right]$$

Several characteristic details are observed when we consider this three-dimensional plot. First of all, this is the general shape resembling amphitheatre. The crest maximal height is almost at the zero meridian, and the amplitude decreases in both opposite directions. The amphitheatre represents an oval, the contour of which maximally approaches the center from the inside. It is clear that this figure will resemble the auroral oval in the projection onto the ionosphere. The situation changes principally when the boundary conditions are dependent on time (Fig. 3). The solution structure is so that $p_0(t)$ can be considered as an input signal multiplied by the transfer function: $p(t') = G(t) A(L)$.
Fig. 1. Gas pressure relief under the stationary boundary conditions but with the electric field variable in time: (a) $t = 0 \text{ s}$, (b) $t = 1000 \text{ s}$, (c) $t = 2800 \text{ s}$, and (d) $t = 4500 \text{ s}$.

The projection (mapping) of the plasma pressure “hump” onto the ionosphere corresponds to the form and position of the auroral oval. This projection, like the real oval, executes a motion with a change of the convection electric field, and expands with an enhancement of the field.

Fig. 2. A profile (calculated values) of plasma pressure (fig.1b).
Fig. 3. Gas pressure relief (calculated values) under the nonstationary boundary conditions: (a) $t = 0$ s, (b) $t = 1000$ s, (c) $t = 2800$ s, and (d) $t = 4500$ s.

The structure of solution: $P = G^\alpha(t') A^\alpha(R(t))$

$$G = G^\alpha_0 \begin{cases} 1 & t' < 0 \\ 2t'/500 + 1 & 0 \leq t' \leq 500 \\ -t'/500 + 4 & 500 < t' \leq 1500 \\ 1 & t > 1500 \end{cases}$$
The projection (mapping) of the plasma pressure “hump” onto the ionosphere corresponds to the form and position of the auroral oval. This projection, like the real oval, executes a motion with a change of the convection electric field, and expands with an enhancement of the field. The flux density of precipitating electrons at the level of the ionosphere will be $j_{\parallel e} = B^\parallel B^\parallel n dl/B\tau$ [Ponomarev, 1985; Ponomarev, Sedykh, 2006]. The time variation in precipitation during a model substorm is shown in Fig. 4, 5.

Fig. 4. Contour lines of the intensity of the precipitating electron flux density for the nonstationary boundary conditions.
Fig. 5. Contour lines of the intensity of the precipitating electron flux density for the nonstationary boundary conditions.

Fig. 6. Dynamics of aurora from the Polar satellite on January 6, 1998 at 1621-1717 UT [S.I. Solovyev et al., 2003].
MAGNETOSPHERE-IONOSPHERE COUPLING

From the plasma movement equation in the single-liquid approximation: \( \rho \frac{dV}{dt} + \nabla p = \frac{[j x B]}{c}, \)
where \( V \) – is the mass movement velocity, \( \rho \) - plasma density, \( p \) – gas pressure, \( j \) – current density, \( B \) – magnetic field intensity. Having multiplied the equation on \( V \) and taking into account that we can always neglect the inertia power in magnetosphere, we get a very important for the Earth magnetosphere physics ratio: \( V \nabla p = Ej. \) In the left part of it we have the hydrodynamic quantities, and in the right one – electrodynamic ones. The physical sense of this equation is clear. If the gas is moving to the pressure increase (i.e. in the plasma coordinate system happens its compression – the compressor «works»), then \( Ej >0. \) Now we have the consumer of the electric power – MHD compressor. If the plasma is flowing to the side of pressure decrease, then the gas-kinetic power can provide a work over the electric ones. Knowing the propagation of the plasma pressure we can easily the places of MHD-compressor and MHD-generators location magnetosphere. The last ones appeared to be, as a minimum, three (Fig.7). The first one – on the internal border of the plasma layer. This generator works on the gradient of the gas pressure \( \nabla_r p, \) and the two generators, working on the longitude gradients \( \nabla_\lambda p \) (see in details [Ponomarev, 1985]). The generators feed the current systems of Birkeland-Bostrom (BB) of the first and second types [Birkeland, 1913; Bostrom, 1964]. There is more detailed discussion of this question in [Ponomarev, Sedykh, 2006, Geomagnetism & Aeronomy].

The second amphitheatre and a specific corridor between the amphitheatre and the main crest appear in Fig. 3b. The cross section of this spatial pattern along any intermediate contour line is shown in Fig. 8. Figure 8 indicates that the field-aligned currents are directed oppositely on both sides of the corridor. Since the sign of the \( p_g \) gradient changed and that of the \( p_B \) gradient remained unchanged, the double "curtain" of field-aligned currents is formed, which is a characteristic feature of auroral electrojet feeding. The geometry of these electrojets corresponds to that of the BB current loop of the second type.
Fig. 7. Schematic spatial location of the magnetospheric--ionospheric currents[Ponomarev, 1985; Ponomarev, Sedykh, 2006]: (a) the system of feeding the meridional currents, (b) the system of feeding the latitudinal currents, and (c) the equivalent scheme of the magnetospheric--ionospheric current circuit. The currents are shown by thin solid lines; electric fields, by open arrows; and convection, by dashed lines. The wavy line corresponds to the region of bulk charge localization. (G) electric energy generators, (C) MHD compressor, (R) ionospheric sections of the circuit with active load, and (I) corresponding currents.
Fig. 8. Schematic of the section of the gas pressure relief. The section of the “gorge” is represented by “corridors” or channels of magnetospheric MHD-generators, on the walls of which field-aligned currents are generated. The walls of the “corridors” serve as the sources of two bands of field-aligned currents, the direction of which is opposite on different walls. There arises a current configuration corresponding to the well-known Iijima and Potemra scheme. The prototypes of the channels – plasma “corridors” are located close to gas pressure maximum, i.e. maximum of particles precipitations into the ionosphere, where auroral electrojets are located [Sedykh, Ponomarev, 2002, Geomagnetism and Aeronomy].
Figure 9. Layout of the functional blocks in the magnetosphere:
1, 2 - secondary MHD generators that convert compressed gas energy to electric current feeding electrojets in the ionosphere.
3 - MHD compressor that converts electric energy to gas pressure;
4 - MHD generator that converts solar wind kinetic energy to electromagnetic energy (when Bz<0)[Ponomarev et al., 2006];
The density of the field-aligned current from the high-latitude ionosphere (fig.10) was calculated using the formula:

$$j_r = \left[ \frac{\partial J_\lambda}{\partial \lambda} + \cos \theta \cdot J_\theta + \sin \theta \cdot \frac{\partial J_\theta}{\partial \theta} \right]/R_0 \sin \theta;$$

where $J_\lambda = \Sigma_p E_\lambda$; $J_\theta = \Sigma_p E$; $\Sigma_p = \left( \frac{e^2 N_e}{M_i} \right) \int \nu_{in}/(\omega_{iB}^2 + \nu_{in}^2) \, dz$;

$N_e = \left( \frac{j_e}{H \delta \epsilon} \alpha \right)^{1/2}$; Here $\delta \epsilon$ is expressed in erg/cm$^2$; recombination coefficient ($\alpha$), in cm$^3$ s$^{-1}$; and $H$ (the dynamo layer thickness), in cm; $e$ is the electron charge, $M_i$ is the ion mass, $\omega_{iB}$ is the ion gyrofrequency, and $\nu_{in}$ - the ion--neutral collision frequency.

On the other hand, the gas pressure gradient is responsible for the bulk density of the current: $j_\perp = c \left[ B \times \nabla p_g \right] / B^2$, and the field-aligned current is determined by divergence of the bulk current, it is clear that the plasma pressure distribution is completely responsible for the field aligned current pattern (fig.10) and, consequently, for the scheme of feeding of the ionospheric current systems. The expression for the field-aligned current [Ponomarev, 1985]:

$$j_{||} = c B^I \oint_{0} \left\{ [\nabla p_g \times \nabla p_B] \cdot B/p_B B^3 \right\} \, dl$$

the expression for $j_{||}$ is well-known Vasyliunas-Tverskoy’s expression for FAC; where $B^I$ is the magnetic field strength in the ionosphere, the integral is taken over the entire flux tube from the equator to the ionosphere, and $p_B$ is the magnetic pressure. It is clear that the integrand is proportional to the sine of the angle between the contour lines $p_g = \text{const}$ and $p_B = \text{const}$.
Figure 10 shows the distribution of the field-aligned currents from the ionosphere (the upper panels) and magnetosphere (the middle panels) and the combined distributions (the lower panels) in the projection on the northern polar region. It is evident that the general construction of the field-aligned currents corresponds to the known empirical pattern [Iijima, Potemra, 1978].

The combination was performed in the following way. If the field-aligned current amplitude in a given zone was lower (higher) than the threshold density dependent on the noise level, index 0 (1) was attached to this amplitude. Both field-aligned current patterns were numbered in such a way and were subsequently multiplied. If the area occupied by unities in the ionospheric, magnetospheric, and combined patterns are denoted by $S_i$, $S_m$, and $S_c$, the combination quality criterion will be the number:

$$K\% = 100\left(\frac{S_i - S_c}{S_i + S_c} + \frac{S_m - S_c}{S_m + S_c}\right)$$

$K$ is mostly about 2% and only sometimes reaches 5%.

Thus, we can state that the field-aligned currents, which resulted from the divergence of the ionospheric surface currents only due to nonuniformity of the ionospheric conductivity, and the magnetospheric currents, which resulted from the convergence of the contour lines of the gas and magnetic pressures, proved to be in rather good natural agreement [Sedykh, Ponomarev, 2004].

Unfortunately, direct observations of plasma distribution in the magnetosphere are faced with large difficulties, because pressure must be known everywhere in the plasma sheet at high resolution, which in situ satellites have been unable to provide. As shown by Waters et al. (2001, Geophys. Res. Lett.), a map of global field-aligned currents can be constructed with hourly resolution using magnetometer data from the Iridium System consisting of 66 satellites in circular polar orbits. Modelling of distribution of plasma pressure (on $\sim 3$-$10$ Re) is very important, because, though multisatellite projects actively develop (and very useful), but the data from 66 satellites would be a very expensive mission.
Fig. 10. Results of calculations of the field-aligned currents ((a) $t = 0 \text{ s}$, (b) $t = 1000 \text{ s}$, and (c) $t = 2800 \text{ s}$) generated in the ionosphere (the upper panels) and magnetosphere (the middle panels), as well as the comparison (compatibility) of these currents (the lower panels); (1) and (2) the zones of inflowing and outflowing currents, respectively.

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Two systems of FAC:
1. Field-aligned currents, which resulted from the divergence of the ionospheric surface currents only due to nonuniformity of the ionospheric conductivity (the upper panels).
2. Field-aligned currents as divergence of the magnetospheric bulk currents; (resulted from the convergence of the contour lines of the gas and magnetic pressures) - the middle panels.
Acknowledgment. The work was done within the framework of the grant MK-3697.2008.5.

References.