Nonlinear Langmuir wave processes in type III solar radio bursts

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Outline

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6. Summary
1.1 Type III radio bursts

- Electron beams propagate outward from the sun along magnetic field lines generating Langmuir waves.
- Radio waves are produced at $f_p$ and $2f_p$ and have a characteristic “L” shape.
- Langmuir waves are observed in type III source regions at 1 AU.

Spectrogram recorded by STEREO A and B on 2011 February 26. Multiple type III event are observed.

Image from http://swaves.gsfc.nasa.gov/
1.2 Type III source regions

- Langmuir waves are generated by the bump-on-tail instability.
- Understanding what Langmuir processes occur is crucial for understanding how radio waves are produced.
- The figures below show two examples of Langmuir events in type III source regions.
2.1 Wave packets and collapse threshold

- Wave packet collapse is the process by which localized Langmuir waves shrink in volume and increase in field strength.
- Wave packet collapse occurs when nonlinear self-focusing exceeds linear dispersion.
- The collapse threshold is defined as: \( \Theta \approx W_{\text{max}} (l/\lambda_D)^2 \)
  where \( W(r) = \frac{\epsilon_0 |E(r)|^2}{4n_e k_B T_e} \) is the normalized energy density.
- Both theoretical work and 3D simulations show that \( \Theta \geq 230 \) is required for collapse to occur.

[e.g., Robinson, 1997; Graham et al., PoP, 2011a,b]
2.2 Previous observations and motivation

- Early observations of localized Langmuir packets were seen as evidence for collapse. [e.g., Gurnett et al., 1981]
  
  Figure from Gurnett et al., 1981. Waveform observed by Voyager 1 at Jovian foreshock.

- But subsequent analyses of Voyager and Ulysses data show that the fields are too weak for collapse to occur. [e.g., Cairns and Robinson, 1992a, 1995; Nulsen et al., 2007]

- However, recent work has argued for collapse based on STEREO data. [e.g., Thejappa et al., 2012a, b, c]

- STEREO/TDS records waveforms from three orthogonal antennas, providing more information on the structure of Langmuir waveforms, motivating us to reinvestigate collapse.
2.3 Estimating the collapse threshold

- Quantities $l/\lambda_D$ and $W_{\text{max}}$ are calculated by assuming STEREO transits the wave packet at characteristic length $l$ from the center.

- $W_{\text{max}}$ is calculated by extracting the electric field envelope (red) from the three perpendicular E fields.

- $l/\lambda_D$ is calculated from the width of the electric field envelope, the solar wind speed, and $T_e = 1.5 \times 10^5$ K.

- We assume a field structure of

$$E(r) = \frac{-a}{(r^2 + l^2)^{\frac{3}{2}}} (l^2 + r^2 - 2x^2, -2xy, -2xz)$$

[Cairns and Robinson, 1992a,1995]
2.4 Collapse threshold estimates

- Characteristic length $l$ and energy density $W(l)$ from STEREO/TDS data are compared with collapse threshold $\Theta$.
- 167 Langmuir packets from eight different type III bursts are considered.
- None of the Langmuir packets identified exceed the collapse threshold.
- For most packets the peak energy density is over an order of magnitude too small for collapse to occur.

[Graham et al., JGR, 2012]
2.5 Detailed fitting

- Collapse theory and simulations show wave packets to be well fitting by potentials

\[ \Phi_L(r) = \frac{ax + iby + c}{l^2 + r^2} \quad \Phi_G = (ax + iby + c) \exp \left( -\frac{r^2}{2l^2} \right) \]

Lorentzian \quad \text{Gaussian}

- We fit the potentials to the three orthogonal fields simultaneously.

- We fit the fields given by \( E = -\text{grad} \Phi \) to the observed waveforms.

2.6 Examples of detailed fitting

- The Gaussian potential provides the better fit to the data. Very good Gaussian fits are found.

Fits $\Theta = 12$

- Observed fields
- Lorentzian fit
- Gaussian fit

$\Theta = 0.67$  

[Graham et al., JGR, 2012]
2.7 Results of detailed fitting

- Detailed fitting is applied to packets observed on 2011 February 26.
- Good agreement between good fits and approximate method are found.
- When good fits are found and $l_i/l < 2$ the packets are below threshold and collapse cannot occur.
- $\Theta > 230$ only when fits imply $l_i/l > 3$. These estimates are unreasonable.

[Approximate method - Good fits - Average fits - Poor fits]

[Graham et al., JGR, 2012]
3.1 Langmuir Eigenmodes: Theory

• For a parabolic density well Langmuir eigenmodes have the form: [Ergun et al., 2008]

\[ E(x, t) = \sum_n A_n H_n(Qx) e^{-Q^2 x^2/2} e^{i(k+\Delta k)x-i(\omega+\Delta \omega_n)} \]

• These eigenmodes grow in preexisting density wells. [e.g., Ergun et al., 2008; Hess et al., 2010]

• Unlike collapsing wave packets, eigenmodes do not need to exceed a threshold field strength.
3.2 Langmuir eigenmodes fits

- Fits to eigenmode theory agree well with data for most localized waveforms.
  
  [Ergun et al., 2008]

- Eigenmodes provide a better explanation for localized waves than wave packet collapse.

Images from Graham et al., ApJL, 2012a. Both packets are below the collapse threshold.
4.1 Electrostatic Decay: Theory

- Langmuir waves generated by electron beams have wave number \( k_b = \frac{\omega_p}{v_b} \).
- These Langmuir waves can decay into backward propagating Langmuir waves: \( L \rightarrow L' + S \).
- By assuming the linear dispersion relations

  \[ \omega_L = \omega_p + \frac{3v_e^2k^2}{2\omega_p} \quad \text{and} \quad \omega_S = v_s k \]

  the wave numbers are:

  \[ k_L = k_b, \]
  \[ k_{L'} = -k_b + k_0, \]
  \[ k_S = 2k_b - k_0, \]

  where \( k_0 = 2\omega_p v_s / 3v_e^2 \).
4.2 Doppler-shifted frequencies

- Langmuir waves are convected past STEREO at $v_{sw}$.
- Therefore STEREO will observe waves at Doppler-shifted frequencies:
  \[
  f^d_L = f_p \left( 1 + \frac{3v_e^2}{2v_b^2} + \frac{v_{sw} |\cos \theta|}{v_b} \right),
  \]
  \[
  f^d_{L'} = f_p + f_p \left( \frac{3v_e^2}{2v_b^2} - \frac{2v_s}{v_b} + \frac{2v_s^2}{3v_e^2} \right) - f_p \left( \frac{1}{v_b} - \frac{2v_s}{3v_e^2} \right) v_{sw} |\cos \theta|.
  \]
- The predicted Doppler-shifted frequency difference is:
  \[
  \Delta f^d_{LL'} = f^d_L - f^d_{L'} = 2f_p \left( \frac{1}{v_b} - \frac{v_s}{3v_e^2} \right) (v_s + v_{sw} |\cos \theta|) .
  \]
- For ion-acoustic waves, the predicted Doppler-shifted frequency is:
  \[
  f^d_S = \left| \Delta f^d_{LL'} \right| \quad \text{[Cairns and Robinson, 1992b; Henri et al., 2009]}
  \]
4.3 Langmuir/z-mode waves

- In a magnetized thermal plasma Langmuir waves connect to the magnetoionic z-mode wave to form the Langmuir/z-mode wave.  
  [Willes & Cairns, 2000; Layden et al., 2011]

- Langmuir portion of the mode is electrostatic ($E$ parallel to $B_0$).

- Z-mode portion is electromagnetic ($E$ perpendicular to $B_0$).

- $F = \frac{E_{\text{perp}}^2}{E_{\text{tot}}^2}$ is the proportion of perpendicular energy density to total energy density.  
  [Graham et al., 2012, JGR, submitted]
4.4 Decay of Langmuir/z-mode waves

• For $k_b > k_0$ Langmuir waves decay into Langmuir-like waves, implying small $F$.
• For $k_b \leq k_0$ Langmuir waves decay into z-mode-like waves, implying large $F$.

• For decay of Langmuir waves to z-mode-like waves the Doppler-shifted frequency difference is:

$$\Delta f_{LC}^d = \frac{3f_p v_e^2}{2v_b^2} + \frac{f_p v_{sw} |\cos \theta|}{v_b}.$$

• Z-mode waves can form for $k_b > k_0$ if multiple decays occur (i.e., an ES backscatter followed by a decay to z-mode waves).
4.5 Doppler-shifted frequencies versus $v_b/c$

- Plot of Doppler-shifted frequencies versus $v_b/c$ for nominal solar wind conditions
- Dashed line is $f_p$.
- Z-mode waves have frequencies near $f_p$, so are difficult to distinguish from $f_{L'd}$.  

- $v_b/c$ is estimated by assuming electrons travel at constant speed along a Parker spiral.
4.6 Event selection

• We analyse Langmuir waveforms in type III source regions between 2009 June and 2012 February; a total of 596 events were selected.

• We also divide events into F < 0.2 and F > 0.2, corresponding to weak and strong perpendicular fields.

• Figure of F versus $v_b/c$ for Langmuir events with multiple distinct spectral peaks (86 events for F < 0.2, 145 events for F > 0.2).

• weaker $E_{\text{perp}}$ are generally observed at lower $v_b/c$. 
4.7 Example of electrostatic decay (F < 0.2)

- STEREO events on 2011 January 22.
- Panels: waveforms of $E_{\text{par}}$, wavelet transforms, and power spectra.
- Left: before ES decay.
- Right: during ES decay.
- Observed and expected frequency differences agree (360±80Hz versus 300±90Hz).


4.8 ES decay (F < 0.2) – general analysis

- Expected $\Delta f$ calculated using: 
  \[
  \Delta f^{d}_{LL'} = 2f_p \left( \frac{1}{v_b} - \frac{v_s}{3v_e^2} \right) (v_s + v_{sw}|\cos \theta|)
  \]

- Observed and expected $\Delta f$ agree well.

- When intense ion-acoustic waves are observed then $\Delta f = f_s$ as expected for ES decay.

- These results provide strong evidence for ES decay in type III source regions.

4.9 Decay example ($F > 0.2$)

- Example of a decay event with strong perpendicular fields.
- Higher frequency peak is $E_{\text{par}}$.
- Lower frequency peak is $E_{\text{perp}}$.
- Observed frequency difference is consistent with decay to low-$k$ Langmuir-z mode waves.

4.10 Decay to Langmuir/z waves (F > 0.2)

- Expected $\Delta f$ calculated using: $\Delta f_{LC}^d = \frac{3 f_p v_e^2}{2 v_b^2} + \frac{f_p v_{sw} |\cos \theta|}{v_b}$.
- Observed and expected $\Delta f$ agree well.
- When intense $S$ waves are observed $\Delta f = f_s$, as expected for ES decay to $z$-mode waves.
- These results provide strong evidence for decay to Langmuir/z-mode waves.

4.11 ES decay and decay to Langmuir/z waves

- Power spectra at (a) $v_b/c = 0.05$, (b) $v_b/c = 0.10$, (c) $v_b/c = 0.18$.
- For $v_b/c < 0.1$ Langmuir waves undergo a single ES decay to backscattered Langmuir waves.
- For $v_b/c > 0.1$ Langmuir waves generally decay to z-mode waves at low $k$.
- For $v_b/c \sim 0.1$ three peaks similar to (b) are commonly observed.

- $E_{\text{par}}$
- $E_{\text{perp}}$

5. Discussion

• Approximately 35% of observed events appear to be localized, suggesting Langmuir eigenmodes.

• Approximately 40% of observed events are likely to be decay events.

• Langmuir eigenmodes can produce radio waves at \( f_p \) and \( 2f_p \) via the antenna mechanism [Malaspina et al., 2010, 2012].

• Transverse waves can be produced by coalescence \( L + L' \rightarrow T(2f_p) \), EM decay \( L \rightarrow T(f_p) + S \), or mode conversion of Langmuir/z-waves.
6. Summary

- Localized Langmuir waves (~35% of TDS events) are inconsistent with collapsing wave packets but are generally consistent with eigenmodes of density wells.

- ES decay of Langmuir-like waves to Langmuir-like and z-mode-like waves is commonly observed (~40% of TDS events).

- Z-mode waves near k = 0 are commonly observed (~25% of TDS events).

- Both Langmuir eigenmodes and ES decay may be important in producing the radio waves observed in type III bursts.

References