The Effect of Irregularities on the Direct Current

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Manuscript Title: Effect of Electrojet Irregularities on DC Current Flow

Dear Stephan:

I am pleased to accept the above manuscript for publication in the Journal of Geophysical ... 

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Special thanks to A. Richmond, Editor of JGR
Electrojet Irregularities

- occur in the E-region ionosphere at altitudes \( \approx 90–120 \) km,
- at the magnetic equator, in the auroral zone, sometimes at mid latitudes
- mainly field-aligned density variations seen by radars as Bragg scattering
- typical wavelengths 1–30 m
- explained by the Farley-Buneman instability
  \( \rightarrow \) ion and e\(^-\) velocity difference exceeds ion sound velocity
- electric current mainly a Hall current?
Heating of the Ionosphere

IS radar (EISCAT) measures electric field $E_0$, $T_e$, and $T_i$. Whenever $|E_0| >$ a threshold ca 30 mV/m, then

- Electron heating at altitudes
  $\approx 98–115$ km
- $T_e$ increases from
  $\approx 300$ K up to
  $\approx 2000$ K
- Ion heating at altitudes above
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Collisional Ion Heating

- Ion heating due to ion-neutral collisions and imposed $E_0$
- Ion-neutral collisions demagnetize ions, $45^\circ \angle$ between ion drift and $E \times B$ at altitude $\approx 130$ km
- Dissipative Pedersen current $j_P$ closes Birkeland current
- Heating rate $j_P \cdot E_0$
- Magnetic effect of currents equivalent to convergence of Poynting flux $S$, $\nabla \cdot S = -j_P \cdot E_0$
- Transfer of electromagnetic energy from (far) above into the polar ionosphere
- Ultimately, the neutral upper atmosphere is heated
How about the Electron Heating?

- $e^{-}$ collision frequency $\nu_e \ll \Omega_e$ $e^{-}$ gyrofrequency

\[
E_0 = -\frac{m}{e} \begin{pmatrix}
\nu_e & -\Omega_e & 0 \\
+\Omega_e & \nu_e & 0 \\
0 & 0 & \nu_e
\end{pmatrix} v_0
\]  

- zero order $e^{-}$ drift $v_0 \approx E_0 \times B/B^2$

1998: We (don’t need no ... theory and) postulate that in the presence of irregularities

- the mean electron drift $\langle v \rangle \neq v_0$
- the mean current $\langle j \rangle$ is partially a Pedersen current, $\langle j \rangle \cdot E_0 > 0$ even in the lower $E$ region

Plan: parameterize the effective $\sigma^*_P(|E_0|)$ using EISCAT data, to improve conductivity models for AMIE ...
The Plan

- assume that a density spectrum $\left\langle |N_1(k, \omega)|^2 \right\rangle$ is given (by theory, simulation ...) or has been measured
- calculate the mean current $\langle j \rangle$ for this density spectrum and then the external (magnetospheric) power input $\langle j \rangle \cdot E_0$
- calculate also the mean Joule heating rate $\langle j \cdot E \rangle$ (wave heating?)
Zero and first order quantities

Current

\[ j(r, t) = e (N(r, t) \mathbf{V}(r, t) - n(r, t) \mathbf{v}(r, t)) \]  \hspace{1cm} (2)

\( N(r, t) \) and \( \mathbf{V}(r, t) \) ion density and velocity
\( n(r, t) \) and \( \mathbf{v}(r, t) \) e\(^-\) density and velocity

\[ N(r, t) = N_0 + N_1(r, t) \]
\[ \mathbf{V}(r, t) = \mathbf{V}_0 + \mathbf{V}_1(r, t) \]
\[ n(r, t) = n_0 + n_1(r, t) \]
\[ \mathbf{v}(r, t) = \mathbf{v}_0 + \mathbf{v}_1(r, t) \].
Mean quantities

\[ \langle f(\mathbf{r}, t) \rangle = \frac{1}{V} \int_V d(\mathbf{r}) \frac{1}{T} \int_T df(\mathbf{r}, t). \]

The mean current

\[ \langle \mathbf{j}(\mathbf{r}, t) \rangle = e(N_0 \mathbf{V}_0 - n_0 \mathbf{v}_0 \]

\[ + \langle N_1(\mathbf{r}, t) \mathbf{V}_1(\mathbf{r}, t) \rangle - \langle n_1(\mathbf{r}, t) \mathbf{v}_1(\mathbf{r}, t) \rangle \] (3)

is affected by correlations between densities and velocities.
Fourier transform

First order ion current

\[
\langle j_{i1}(r, t) \rangle = \left( \frac{1}{2\pi} \right)^4 \frac{e}{VT} \int \int \int d(k) d\omega \langle \Re (V_1(k, \omega) N_1^*(k, \omega)) \rangle
\]  \hspace{1cm} (4)

and similarly for the e\(^-\) current

\[
\langle j_{e1}(r, t) \rangle = -\left( \frac{1}{2\pi} \right)^4 \frac{e}{VT} \int \int \int d(k) d\omega \langle \Re (v_1(k, \omega) n_1^*(k, \omega)) \rangle
\]  \hspace{1cm} (5)

Next establish relation between first order velocities and densities.
Continuity and Momentum Equations

\[ 0 = -i\omega N_1 (k, \omega) + i k \cdot V_1 (k, \omega) N_0 \] (6)

\[ 0 = -in_1 (k, \omega) (\omega - k \cdot v_0) + i k \cdot v_1 (k, \omega) n_0 \]

\[-i\omega V_1 (k, \omega) = \frac{e}{M} E_1 (k, \omega) - i k \frac{\kappa T_i N_1 (k, \omega)}{M N_0} - \nu_i V_1 (k, \omega) \]

\[ \frac{e}{m} (v_1 (k, \omega) \times B_0) = -\frac{e}{m} E_1 (k, \omega) - i k \frac{\kappa T_e n_1 (k, \omega)}{m n_0} - \nu_e v_1 (k, \omega) \]
Assumptions

- zero $e^-$ mass
- no effect of the magnetic field on the ions, $V_0 = 0$
- quasi-neutrality, $n_0 = N_0$ and $n_1 = N_1$
- $k$ component parallel to $B$ negligible
- imaginary part of $\omega$ small compared to real part
Dispersion Relation for Farley-Buneman instability

\[(\omega - k \cdot \mathbf{v}_0) = \frac{M}{m} \left( \frac{\omega(i\omega - \nu_i)}{k^2} - iC_s^2 \right) \left( \frac{\nu_e(k_x^2 + k_y^2)}{\Omega_e^2 + \nu_e^2} + \frac{k_z^2}{\nu_e} \right) \quad (7)\]

\[\omega_r = \frac{k \cdot \mathbf{v}_0}{1 + \Psi_0} \quad (8)\]

where \(\Psi_0\) has the usual meaning:

\[\Psi_0 = \frac{M \nu_i}{m k^2} \left( \frac{\nu_e(k_x^2 + k_y^2)}{\Omega_e^2 + \nu_e^2} + \frac{k_z^2}{\nu_e} \right) \approx \frac{\nu_e \nu_i}{\Omega_e \Omega_i} \quad (9)\]
Mean Current

\[
\langle \mathbf{j}(\mathbf{r}, t) \rangle = -eN_0 \mathbf{v}_0 \\
+ \left( \frac{1}{2\pi} \right)^3 \frac{e}{VT} \iiint d(\mathbf{k}) \mathbf{A} \frac{\mathbf{k} \cdot \mathbf{v}_0}{1 + \psi_0} \frac{\langle |N_1(\mathbf{k}, \omega_r)|^2 \rangle}{N_0}
\]

(10)

Vector \( \mathbf{A} \) has components:

\[
A_x = \frac{k_x}{k^2} + \frac{M}{m} \left( \frac{\nu_i}{k^2} \right) \frac{k_x \nu_e - k_y \Omega_e}{\Omega_e^2 + \nu_e^2}
\]

\[
A_y = \frac{k_y}{k^2} + \frac{M}{m} \left( \frac{\nu_i}{k^2} \right) \frac{k_y \nu_e + k_x \Omega_e}{\Omega_e^2 + \nu_e^2}
\]

(11)

\[
A_z = \frac{k_z}{k^2} + \frac{M}{m} \left( \frac{\nu_i}{k^2} \right) \frac{k_z}{\nu_e}
\]
\[ \langle \mathbf{j} \rangle \cdot \mathbf{E}_0 \approx \frac{1}{VT} \left( \frac{1}{2\pi} \right)^3 M \nu_i \int \int \int d(k) \frac{(k \cdot v_0)^2}{k^2(1 + \Psi_0)} \frac{\langle |N_1(k, \omega_r)|^2 \rangle}{N_0} \] (12)
Mean Joule Heating Rate

\[ \langle j \cdot E \rangle = \langle j \rangle \cdot E_0 + \langle E_1(r, t) \cdot j_1(r, t) \rangle \] (13)

Split \( E_1(r, t) \cdot j_1(r, t) \) into

\[ l_1 = eE_1(r, t) \cdot (N_1(r, t)V_0 - n_1(r, t)v_0) \]
\[ l_2 = eE_1(r, t) \cdot (V_1(r, t)N_0 - v_1(r, t)n_0) \] (14)

\( l_1 \) affected by correlations between electric field and densities, \( l_2 \) by correlations between electric field and velocities.
Fourier Transform and Averaging

\[ L_1 = - \left( \frac{1}{2\pi} \right)^3 \frac{M\nu_i}{VT} \int d(k) \frac{(k \cdot v_0)^2}{k^2 (1 + \Psi_0)} \frac{\langle |N_1(k, \omega_r)|^2 \rangle}{N_0} = - \langle j \rangle \cdot E_0 \] (15)

and

\[ L_2 = \left( \frac{1}{2\pi} \right)^3 \frac{M\nu_i}{VT} \int d(k) \frac{(k \cdot v_0)^2}{k^2 (1 + \Psi_0)} \frac{\langle |N_1(k, \omega_r)|^2 \rangle}{N_0} = + \langle j \rangle \cdot E_0 \] (16)

\[ L_1 + L_2 = 0 \]
Wave Heating?

- Average wave heating $\langle j_1 \cdot E_1 \rangle = 0!$
- External power input $\langle j \rangle \cdot E_0 = \langle j \cdot E \rangle$ mean Joule heating
- Irregularities affect the DC current, and this alone accounts for the $e^-\text{ heating}$
Summary

a) dB
   E0 < Eth
   Ion Pedersen current
   dB = 0
   E0 < Eth
   H ~ 125 km

b) dB
   E0 > Eth
   Ion Pedersen current
   dB = 0
   E0 > Eth
   H ~ 105 km
   Electron Pedersen current
Conclusions

- Irregularities affect the perpendicular DC
- The ionospheric Pedersen conductivity is effectively non-linear, it depends on the electric field
- Plasma is transported anomalously along \( \mathbf{E}_0 \), eg from the bright to the black aurora (this might explain why auroral arcs can exist a long time)
- The velocity difference between ions and \( \text{e}^- \) is the microphysical cause of the FB instability,
- but the free energy for maintaining a stationary turbulent state is external electromagnetic energy.
- there is no “wave heating” in irregularities, \( \langle \mathbf{j}_1 \cdot \mathbf{E}_1 \rangle = 0 \)
Questions/Outlook

- complete the original plan, $\sigma_P^*(|E_0|)$, using data
- can a corresponding generator be found, for example at the magnetopause?
  - experimentally, with Cluster data?
  - theoretically, eg with lower hybrid waves/irregularities/turbulence
- parallel to $B_0$ waves/irregularities don’t affect the DC (to first order), rather a quasi-stationary $E_\parallel$ is set up
- theoretical prove that this is actually occurring?
- like closure of $j_\parallel$ also $E_\parallel$ causes a divergence of the downward Poynting flux, and this powers the aurora!
- (the velocity difference between ions and electrons due to $j_\parallel$ provides free energy for certain microinstabilities, but it does not provide any significant energy to the aurora)