Övningstentamen för Rymdfysik I och Rymdfysik MN1
2001-11-21

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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well.
Time: 1000 – 1500
Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet.

1. Here follows a set of qualitative questions, each of which should be answered in perhaps 5–15 lines of text, possibly an equation or two and maybe a figure.

   (a) What is an ionosphere? (1 p)

   (b) What is a ”frozen-in” magnetic field? (1 p)

   (c) What is the heliopause? (1 p)

   (d) Why don’t satellites just fall down to the ground? (1 p)

   (e) What is the aurora? What does it look like, and why? What is the altitude of the visible auroral emissions? Does it have any association to geomagnetic substorms? Does it have anything to do with the solar cycle? (2 p)

   (f) What is the solar activity cycle? How long is it, what happens on the sun during it, what effects does it have on the Earth? (2 p)

   (g) Describe the non-relativistic motion of a charged particle in a dipole magnetic field. Three characteristic periodicities are associated with this motion. Which are they and why do they occur? (2 p)

2. The total mass launched by a rocket can be written

$$M = m_{\mathrm{p}} + m_{\mathrm{f}} + m_{\mathrm{s}}$$

   where m_{p} is the payload that we actually want to put into orbit, m_{f} is the fuel and m_{s} is the structural mass, i.e. the mass of the rocket itself.

   (a) Why is it at all good to divide a rocket into several stages? (1 p)

   (b) Show that dividing the rocket into two stages gives an additional Δv

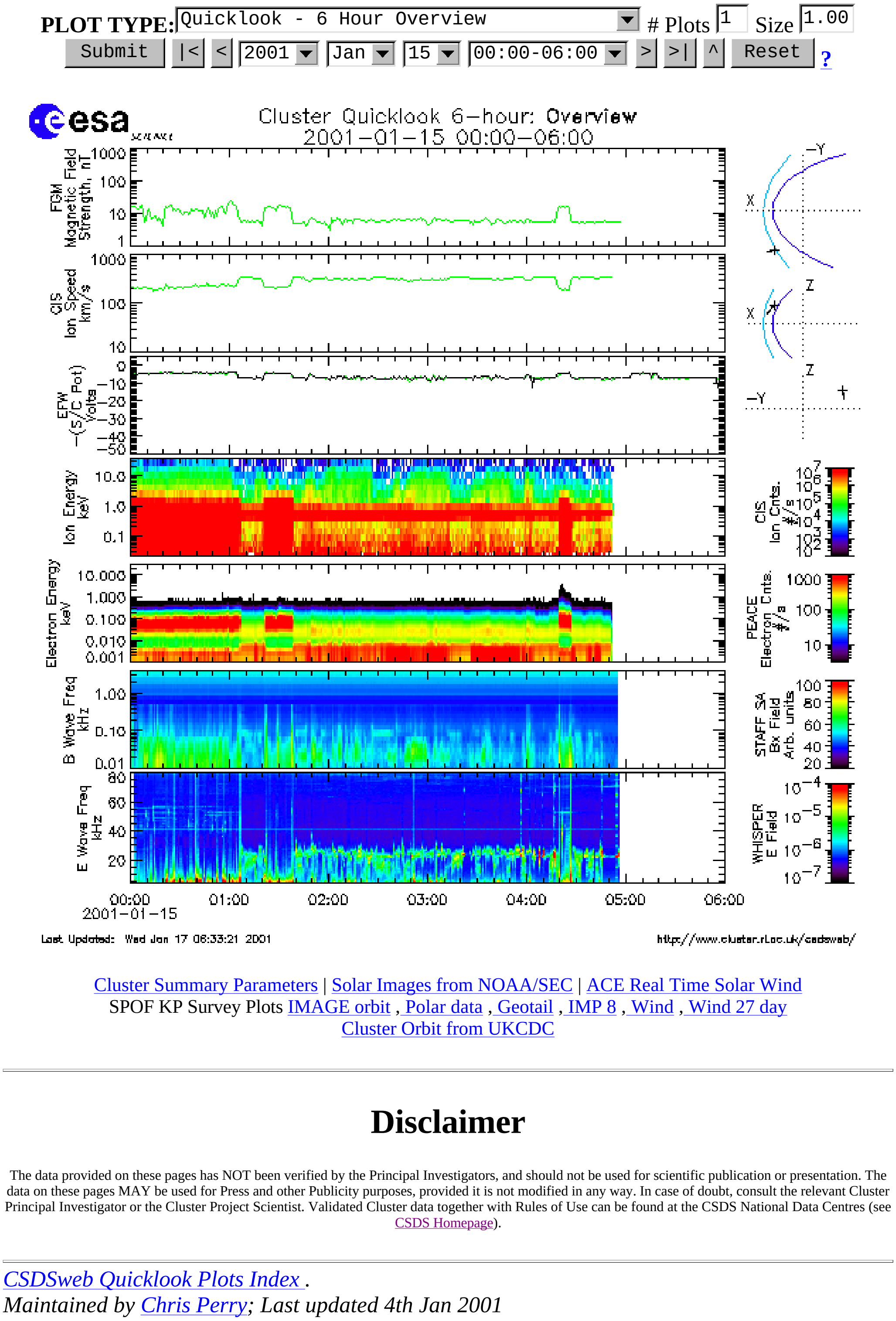
$$(\Delta v)_{\mathrm{bonus}} = v_{\mathrm{e}} \ln \frac{1 + \frac{m_{2\mathrm{f}}}{m_{2\mathrm{s}} + m_{\mathrm{p}}}}{1 + \frac{m_{2\mathrm{f}}}{m_{1\mathrm{s}} + m_{2\mathrm{s}} + m_{\mathrm{p}}}}$$

   as compared to using the same fuel and structure mass in one single stage. (2 p)

   (c) For a rocket launch with $m_{\mathrm{p}} = M/50$ and $m_{\mathrm{s}} = M/10$, how much does Δv increase by dividing a rocket into two identical stages, while still keeping the total mass constant? Assume that the ratio of fuel to structure mass is the same for each stage and also for the one-stage rocket you compare to. The answer should be given in per cent of the one-stage Δv. (1 p)

   We assume that the rockets burn very quickly, so that the term gt_{burn} in the rocket equation can be neglected.

3. In Figure 1, you find data measured by one of the Cluster spacecraft from December 2000. The small sketches at right in the figure show the orbit of Cluster during the period plotted, with a plus sign denoting the start of the time interval. The Earth is at the origin, the ecliptical plane is the xy-plane, and the Sun is far away in the $+x$ direction. The figure also shows predicted positions of the bow shock and of the magnetopause.



Cluster Summary Parameters | Solar Images from NOAA/SEC | ACE Real Time Solar Wind
SPOF KP Survey Plots IMAGE orbit , Polar data , Geotail , IMP 8 , Wind , Wind 27 day
Cluster Orbit from UKCDC

Disclaimer

The data provided on these pages has NOT been verified by the Principal Investigators, and should not be used for scientific publication or presentation. The data on these pages MAY be used for Press and other Publicity purposes, provided it is not modified in any way. In case of doubt, consult the relevant Cluster Principal Investigator or the Cluster Project Scientist. Validated Cluster data together with Rules of Use can be found at the CSDS National Data Centres (see CSDS Homepage).

CSDSweb Quicklook Plots Index .
Maintained by Chris Perry; Last updated 4th Jan 2001

Figure 1: Data from one of the Cluster spacecraft orbiting the Earth. Plot descriptions from top down: (1) Magnetic field strength B. (2) Bulk flow speed (not thermal speed) of ions. (3) Spacecraft potential – please ignore. (4) Ion energy distribution. Dark shading means many ions at a particular energy, light that there are few. The energy given is the total kinetic energy of each ion detected by the instrument, due to bulk flow as well as to thermal motion. (5) Electron energy distribution (plotted in the same way as the ions). (6) and (7) wave spectra – please ignore.

(a) Which of the curves in the small plots at right is the bow shock – the grey (left) or the black (right) curve? (1 p)

(b) Each boundary separates two regions of space. What regions are separated by the bow shock, and what regions are separated by the magnetopause? (State the names of the regions involved) (1 p)

(c) The prediction thus was that Cluster should not cross any of the two boundaries during this time interval. In reality, the data shows that Cluster crossed one of these boundaries several times during this time period (or, rather, that this boundary passed Cluster several times), while Cluster did not see the other boundary at all. Mark the boundary crossings with arrows in plot (2) from above. (I think there are five crossings – do you agree?) (1 p)

(d) Which boundary did Cluster encounter – the bow shock or the magnetopause? Motivate your answer by describing how the data in each of panels (1), (2), (4) and (5), numbered from top down, support your conclusion. (2 p).

4. (a) Consider 1 keV ions and electrons with pitch-angle 90° in the equatorial plane at a distance of 3.9 R_S and 20.3 R_S from the center of the planet Saturn. This corresponds to the distances to the icy moon Enceladus, which is suspected to expel lots of water, and the large moon Titan, with its dense atmosphere, respectively. How long time does it take for the charged particle mentioned above to drift one complete orbit around Saturn? The magnetic field of Saturn may here be considered to be a dipole field with a a magnetic field strength of 21 μT near the cloudy surface at the equator. (R_S = 60330 km is the radius of Saturn.) (3 p)

   (b) How does the drift velocity compare to the co-rotational velocity enforced by the rotation of Saturn? There is still a ring current that develop around Saturn between about 8 R_S and 16 R_S according to the Voyager 1 and 2 magnetic field measurements. How is this possible? Adopt a mean drift velocity value at 10 R_S, assume the particle density is 0.1 cm^{-3} of the 1 keV particle populations within the entire region. Derive the total current flowing through this region. Assume the region can be considered as a rectangular cross section with 5 R_S thickness (see picture). Saturns' spin period is 10 h 14 mins. (2 p)

5. The icy Jupiter moon Europa has a very thin atmosphere of oxygen (O_2 and O), created mostly by sputtering of the surface ice by energetic particles from the magnetosphere of Jupiter. Models based on this source predicts a surface pressure of some 0.01 - 1 μbar. Some believe that outgassing from an under-ice Europa-worldwide ocean could also contribute to this thin atmosphere in which case its surface pressure would be much higher.

   The Galileo spacecraft arrived to the Jupiter system a couple of years ago to investigate the issue closer, among many other tasks. The onboard HF plasma wave instrument estimated the plasma density to 100 cm^{-3} above the background magnetospheric density (essentially by deriving the density from the plasma frequency) at a distance of 0.4 R_{Eu} from the frozen icy sunlit surface of the moon (R_{Eu} = 1565 km is the radius of Europa). The overexcited and famous scientist Frank Splash rushes into your office and screams EUREKA, I KNOW THE PRESSURE AT THE SURFACE OF EUROPA! He says that he has just conjured up, by his masterful scientific intuition, that the photoionization rate of atomic oxygen during the Galileo measurements near Europa can be estimated to 0.1 $\text{m}^{-3}\text{s}^{-1}$. Unfortunately, he stumbles over a chair and dies before he tells you more !

   (a) A note on one of his papers reveals that the important chemical reactions for the formation of the ionophere of Europa are:

   $O + h\nu \rightarrow O^+ + e^-$ (photoionization)
   $O^+ + O_2 \rightarrow O_2^+ + O$ (charge exchange)
   $O_2^+ + e^- \rightarrow O + O$ (recombination)

   where the two last reactions have reaction rates of 10^{-18} m^3/s and 10^{-13} m^3/s respectively. Calculate the local (where the spacecraft was) equilibrium molecular oxygen density (O_2) assuming quasineutrality. (3 p)

   (b) Assume furthermore that diffusive equilibrium exist throughout the whole atmosphere of Europa, which therefore can be considered as an exosphere, with a constant temperature of 100 K (freezing cold!). The gas can be assumed ideal. The surface gravity acceleration is about 1.3 m/s^2 and can be considered not to change significantly with altitude. Calculate the surface molecular oxygen pressure. Find out what the late Prof. Splash discovered and give a comment on if it is likely that outgassing of liquid water occurs on Europa. [If you didn't manage to calculate the O_2 density in the problem above, then assume a value of 10^9 m^{-3} (which is not necessarily the correct answer to the preceding problem).] (3 p)

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for ”frozen-in” magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm’s law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\| \end{pmatrix} \begin{pmatrix} E_\perp \\ 0 \\ E_\| \end{pmatrix}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_\perp^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu\nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v_F} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_\perp / v_\parallel$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_\mathrm{D}}}{r}$$

Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_\mathrm{e}}}$$

Rocket thrust:

$$T = v_\mathrm{e}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_\mathrm{sp} = \frac{\int T\mathrm{d}t}{m_\mathrm{fuel}g} = v_\mathrm{e}/g$$

The rocket equation:

$$\Delta v = -gt_\mathrm{burn} + v_\mathrm{e}\ln\left(1 + \frac{m_\mathrm{fuel}}{m_\mathrm{vehicle}}\right)$$

Emitted thermal radiation power:

$$P_\mathrm{e} = \varepsilon\sigma A_\mathrm{e} T^4$$

Absorbed solar radiation power:

$$P_\mathrm{a} = \alpha A_\mathrm{a} I_\mathrm{rad}$$