Tentamen för kursen Rymdfysik (1FA255)
2021-12-21

Uppsala universitet
Institutionen för fysik och astronomi
Avdelningen för astronomi och rymdfysik
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Answers should be provided in Swedish or English.
Time: 14:00 - 19:00
Allowed tools: Mathematics Handbook (or equivalent), Physics Handbook, enclosed tables and formula sheets, calculator. A bilingual dictionary, for example English-Swedish or English-German, may also be used.
The exam has two parts:

- **Part A** must be satisfactorily solved in order to pass the course. You do not need to solve part A problems corresponding to examlets you have passed during the autumn 2021 course. Part A is graded by pass/fail.
- **Part B** must be solved if you aim for a grade higher than pass (3 in the Swedish 3-4-5 system, E in ECTS). Grades will depend on the number of points you score on this part.

Part A

1. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.
   i. The geostationary orbit is possible due to a balance between the gravitational pull of the earth and the sun.
   ii. As the gravitation from the Earth decreases as $\exp(-z/H)$, where z is the altitude above ground and H is the scale height (see formula sheet), spacecraft at altitudes above about 100 km move in straight lines.
   iii. Satellites in geostationary orbit need to have a lot of fuel to correct their orbits from the perturbations caused by air friction.
   iv. Consider two homogeneous bodies, of equal volume and with identical material properties, in interplanetary space at similar distance to the Sun, a sphere and a cube. They have no internal heat sources and both of them are in thermal equilibrium. Is it true that the sphere is warmer than the cube?
   v. At perigee, a satellite has its highest speed.

   (b) The ESA mission Swarm consists of three satellites around Earth, two travelling close together at an altitude of 450 km and the third at about 530 km, all in close to circular orbits. What is the difference (in minutes) between the periods of the upper and the lower satellites?

2. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

   i. The magnetic field in the frame of reference of the plasma is close to zero (for processes on sufficiently large scales in time and space).
   ii. If the electric field in a plasma, as measured in its rest frame, is everywhere zero, two particles initially connected by a magnetic field line always will be so.
   iii. At 1 AU, the solar wind energy density is dominated the thermal contribution nK_BT (rather than the magnetic energy density$B^2/2\mu_0$ or bulk kinetic energy density $mnv^2/2$).
   iv. The magnetic pressure arises because of the random motion of magnetized particles.
   v. The typical direction of the interplanetary magnetic field is about 45 degrees from the Sun.

   (b) Draw a simple sketch to illustrate the concept of the "Debye length", in a plasma consisting of positive ions and electrons. Why does the plasma temperature T enter the expression for the Debye length?

3. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

   i. The magnetic moment due to the gyration of a charged particle in a magnetic field is an adiabatic invariant if the magnetic field changes very little over a gyroperiod and gyroradius.
   ii. In a homogeneous magnetic field, all particles of the same species, for example electrons, must have the same gyroradius.
   iii. An electric field exactly perpendicular to the magnetic field (both fields homogeneous and static) causes an electric current to flow in a collisionless plasma.
   iv. For a particle moving in a dipolar magnetic field, its velocity vector is exactly aligned with the magnetic field vector at its mirror point.
   v. In general, the drift period is longer than the bounce period, for particles moving in dipolar fields.

   (b) Describe, in words and an illustration, the motion of a positively charged ion and an electron with pitch angle $\alpha = 45^\circ$ along a magnetic field line that everywhere has a radius of curvature much larger than the electron gyroradius but in some places comparable to or smaller than the ion gyroradius.

4. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

   i. The F layer in the Earth's ionosphere disappears quicker after sunset than does the E layer.
   ii. The main reason for ionization in the Earth's ionosphere is the ionizing radiation (mainly in the EUV range).
   iii. The auroral light is emitted when atoms (sometimes also molecules and/or ions) in the upper atmosphere de-excite after having been excited by electrons in the keV range coming down along the magnetic field lines from the magnetosphere.
   iv. In a substorm, the solar wind proton temperature can rise to very high values, which may lead to spacecraft failures.
   v. The Kp index is based on the number of sunspots in a 3 hour interval.

   (b) Figure 1 shows a simulation describing the solar wind radial velocity (left) and plasma number density (right) in the ecliptic plane one week before the time you write this exam (December 14, 18:00 UT), viewed from above the Sun's north pole. The green (light, if you are colour blind) circle below the Sun marks the Earth.

   i. Many structures in both plots have a spiral shape. Why is that?
   ii. Based on this plot, what do you think about the solar wind near the Earth at the time of this exam? Describe your reasoning?
   iii. Discuss a few space weather phenomena that cannot be predicted by this simulation.

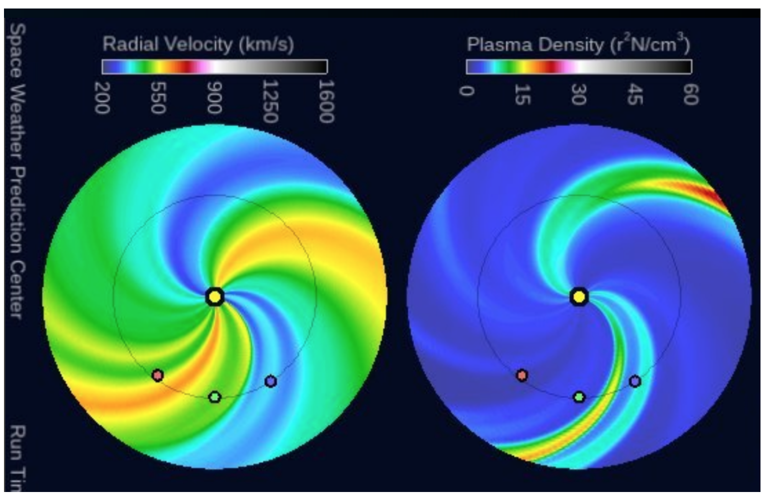

Figure 1: WSA-ENLIL simulation of the solar wind. Image credit: NOAA/SWPC

Part B

5. Figure 2 shows the trajectory (solid curve) of the European Rosetta spacecraft from its launch into space in March 2, 2004 to the final arrival at comet 67P/Churyumov-Gerasimenko in May 22, 2014 (which Rosetta then followed at close distance for more than two years). The reference system used in the plot is an inertial system. Dashed curves mark the orbits of Earth and Mars. Dotted curves mark the orbits of comet 67P (and some short segements of the orbits of two asteroids, Lutetia and Steins). Dates for flybys (marked with solid black circles) of planets and other objects are given. Also shown are thruster firings (open circles), with the Δv of each such maneuvre in parenthesis.

   (a) The labels in the figure show that the first flyby of Earth (denoted Earth 1) occurred close to one year after launch. Is there any way you can conclude from just looking at the figure that there must be about one year from launch to Earth 1? (1 p)

   (b) How much (in per cent) did the total energy (kinetic plus potential) of Rosetta change at the third Earth flyby (Earth 3), if we neglect the thruster firings (Post-Steins, Post-Lutetia)? Did it increase or decrease? Do you think neglecting the thruster firings is a good approximation, or did they significantly change the trajectory? Be sure to motivate your answers and describe your reasoning. (4 p)

6. (a) Draw a sketch of the Earth's magnetosphere, viewed from the dusk side (so that the Sun is to the left of the page). Label the following regions, boundaries and features: Earth's magnetic field, magnetopause, bow shock, magnetosheath, magnetosphere, and stream lines of the solar wind. (1 p)

   (b) Derive an expression for the sub-solar distance from the center of the Earth to the magnetopause, stating any assumptions made. Estimate this distance during a period when the solar wind has a proton density of 5 cm^{-3} and velocity of 400 km s^{-1}. (2 p)

   (c) During periods where the interplanetary magnetic field (IMF) has a strong southward component, the IMF and the Earth's magnetic field can "reconnect". Illustrate, in new sketch similar that you have just done, how IMF field lines in this situation are 'convected' through the Earth's magnetosphere. If the reconnection rate (the rate at which magnetic flux from the IMF is added to the magnetosphere) is larger at the dayside than the corresponding loss at the nightside, what happens to the magnetic field in the magnetotail? (2 p)

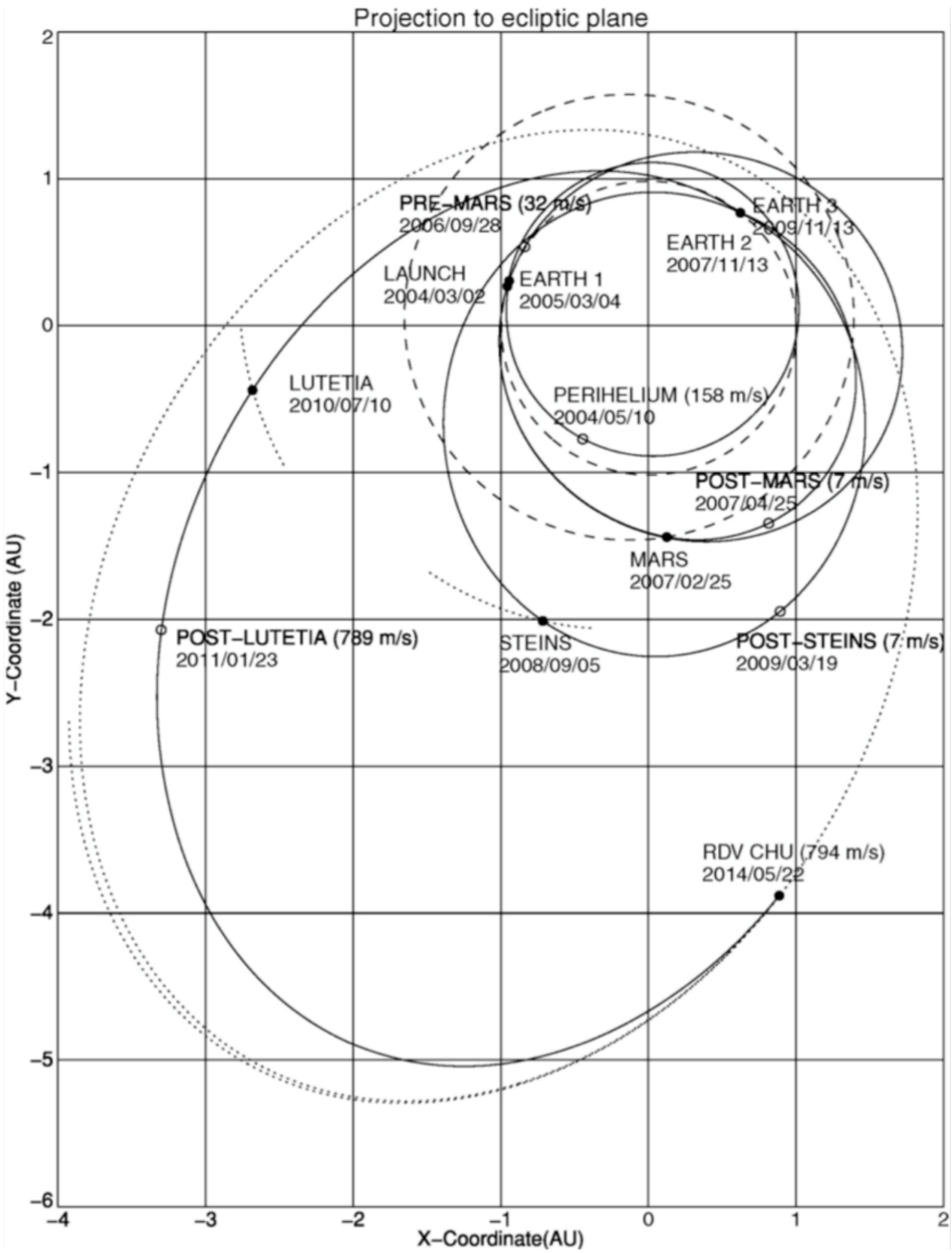

Figure 2: The Rosetta trajectory through the solar system.

7. (a) Show that the kinetic energy of a charged particle moving in a magnetic field, which is constant in time but may vary in space, is constant. (2 p)

   (b) Consider an oxygen ion (O^+) with a kinetic energy of 10 keV and no velocity along the magnetic field, moving in the equatorial plane at a distance of 3 R_E from the center of the Earth. Calculate the two characteristic frequencies defined for this particle (the third one is undefined because of the particle's zero velocity along the magnetic field). Also calculate the gyroradius of the ion. Is it reasonable to describe the motion of the ion as the superposition of two periodic motions as done above? (3 p)

   The geomagnetic field may be taken to be a dipole field with strength 30 μT on the ground at the equator.

8. (a) Derive (from the equation of motion of a neutral gas and an assumption of constant gravitational field) an expression showing why the concentrations of neutral molecules decrease approximately exponentially with increasing altitude, and why the concentration of atomic oxygen (O) decreases slower with altitude than the N_2 density, which in turn decreases slower than the concentration of molecular oxygen (O_2). State explicitly all assumptions you make. (2 p)

   (b) Consider a planet with an atmosphere in hydrostatic equilibrium, i.e. with the neutral gas density profile $n_n(h)$ derived in (a) above. The atmosphere consists of molecular oxygen O_2 and is very tenuous, so the intensity of solar ionizing radiation stays at a constant value I_0 at all altitudes. The only ions created by solar EUV are O_2^+ and the dominating loss term is dissociative recombination. Discussing what assumptions you make, show that the electron density profile $n_e(h)$ will follow an expression of similar mathematical form as the one for the neutral gas, but with twice as high scale height. (3 p)

Lycka till!

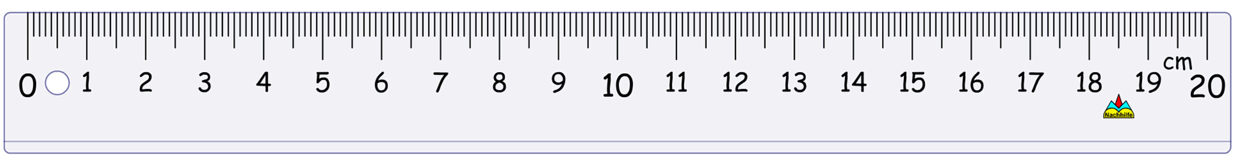

Figure 3: If you did not bring your own ruler, detach this paper and fold it along the upper edge of the image to get a nice working ruler.

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Space Physics Formulas:
Complement to Physics Handbook

Charge density and current density from particle species s:

$$\rho = \sum_s q_s n_s, \qquad \mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Galilean transformations:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{B}' = \mathbf{B}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particle species s:

$$m_s n_s \frac{d\mathbf{v_s}}{dt} = n_s q_s(\mathbf{E} + \mathbf{v_s} \times \mathbf{B}) - \nabla p_s + \text{o.f.}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{o.f.} = -\nabla\left(p + \frac{B^2}{2\mu_0}\right) + \frac{1}{\mu_0}\left(\mathbf{B}\cdot\nabla\right)\mathbf{B} + \text{o.f.}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Dynamic pressure:

$$p_\mathrm{dyn} = \frac{1}{2}nmv^2$$

Condition for ”frozen-in” magnetic field:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Ohm’s law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & -\sigma_\mathrm{H} & 0 \\ \sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_\parallel \end{pmatrix} = \sigma_\mathrm{P}\mathbf{E}_\perp + \sigma_\mathrm{H}\frac{\mathbf{B} \times \mathbf{E}_\perp}{B} + \sigma_\parallel \mathbf{E}_\parallel$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_{\mathrm{c}} = \omega_{\mathrm{c}}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_{\perp}^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu\nabla B$$

Drift motion due to general force **F**:

$$\mathbf{v}_{\mathbf{F}} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_{\perp}/v_{\parallel}$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0}\frac{e^{-r/\lambda_{\mathrm{D}}}}{r}$$

Debye length:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_{\mathrm{p}} = \omega_{\mathrm{p}}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_{\mathrm{e}}}}$$

Rocket thrust:

$$T = v_{\mathrm{e}}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_{\mathrm{sp}} = \frac{\int T\,\mathrm{d}t}{m_{\mathrm{fuel}}g} = v_{\mathrm{e}}/g$$

The rocket equation:

$$\Delta v = -gt_{\mathrm{burn}} + v_{\mathrm{e}}\ln\left(1 + \frac{m_{\mathrm{fuel}}}{m_{\mathrm{payload+structure}}}\right)$$

Total energy of elliptic orbit of semimajor axis a:

$$E = -\frac{GMm}{2a}$$

Kepler's third law:

$$T^2 \propto a^3$$

Emitted thermal radiation power:

$$P_{\mathrm{e}} = \varepsilon\sigma A_{\mathrm{e}}T^4$$

Absorbed solar radiation power:

$$P_{\mathrm{a}} = \alpha A_{\mathrm{a}}I_{\mathrm{rad}}$$

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