Tentamen för kursen Rymdfysik (1FA255)
2021-10-21

Uppsala universitet
Institutionen för fysik och astronomi
Avdelningen för astronomi och rymdfysik
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Answers should be provided in Swedish or English.
Time: 08:00 - 13:00
Allowed tools: Mathematics Handbook (or equivalent), Physics Handbook, enclosed tables and formula sheets, calculator. A bilingual dictionary, for example English-Swedish or English-German, may also be used.
The exam has two parts:

- **Part A** must be satisfactorily solved in order to pass the course. You do not need to solve part A problems corresponding to examlets you have passed during the autumn 2021 course. Part A is graded by pass/fail.
- **Part B** must be solved if you aim for a grade higher than pass (3 in the Swedish 3-4-5 system, E in ECTS). Grades will depend on the number of points you score on this part.

Part A

1. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.
   - i. Rockets do not work in interplanetary space, as they need an atmosphere to push against.
   - ii. The mass needed for a rocket (including fuel) scales approximately exponentially with the velocity increase it provides.
   - iii. To survive the high (10 eV$\sim 10^5$ K) temperature of electrons and ions in the solar wind, interplanetary spacecraft usually need particular cooling arrangements.
   - iv. Consider two homogeneous bodies in interplanetary space with identical material properties and at similar distance to the Sun, a sphere and a very elongated (length $\gg$ radius) cylinder. They have no internal heat sources and both of them are in thermal equilibrium. Is it true that the cylinder will always be warmer than the sphere, whatever way you rotate it in space?
   - v. In a geostationary orbit, the gravity from Earth and the Moon cancel, so satellites always stay at the same position.

   (b) The Molniya orbit is an elliptic orbit with a period of close to 12 hours (that is, half the period of a geostationary satellite whose radius is 42,164 km). If the perigee height of a Molniya orbit is 600 km, at what height is the apogee? The radius of the Earth is 6371.2 km.

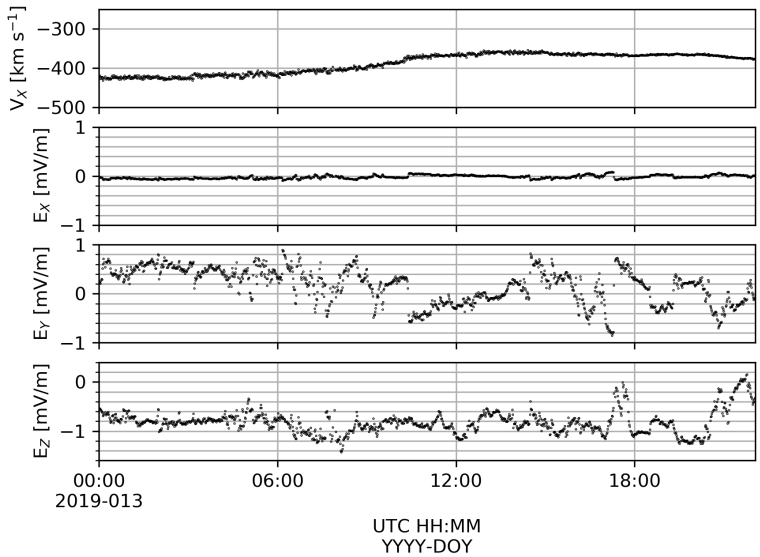

Figure 1: Time series solar wind velocity (V_X) and the electric field in the spacecraft frame (E_X/E_Y/E_Z).

2. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.
   i. Only light with wavelengths longer than infrared (700 nm) can ionize a planetary atmosphere
   ii. Plasmas are expected to be electrically neutral on length scales larger than the Debye length
   iii. The magnetic fields of the magnetized planets are well-described by dipoles, far above their surfaces
   iv. No current can flow through a plasma if quasi-neutrality is to be maintained
   v. MHD dynamo theory can be used to describe the generation (or amplification) of magnetic fields due to plasma motions
   vi. All of the material in the Sun rigidly rotates together, i.e. has the same angular frequency.

   (b) Figure 1 shows solar wind data from a fictional spacecraft that has measured one component of the plasma velocity and three components of the electric field (in the spacecraft frame), expressed in the GSE coordinate system where the X-axis points towards the Sun, and the Y-axis points towards dusk (opposite to the direction of Earth's orbital motion around the Sun). What might you reasonably infer about the other velocity components (V_Y and V_Z) of the solar wind given that $E_X \approx 0$? Estimate a value for B_Z from these data.

3. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.
   i. Distinct cyclotron, bounce and drift motions are the three types of periodic motion made by charged particles in dipolar fields
   ii. The $\boldsymbol{E} \times \boldsymbol{B}$ drift causes a current to flow in a plasma
   iii. Plasmas are generally paramagnetic, i.e. act to increase any external magnetic fields applied to them
   iv. The gyroradius of a particle must be larger than the Debye length
   v. To a good approximation the solar wind does not flow into the magnetosphere, across (through) the magnetopause

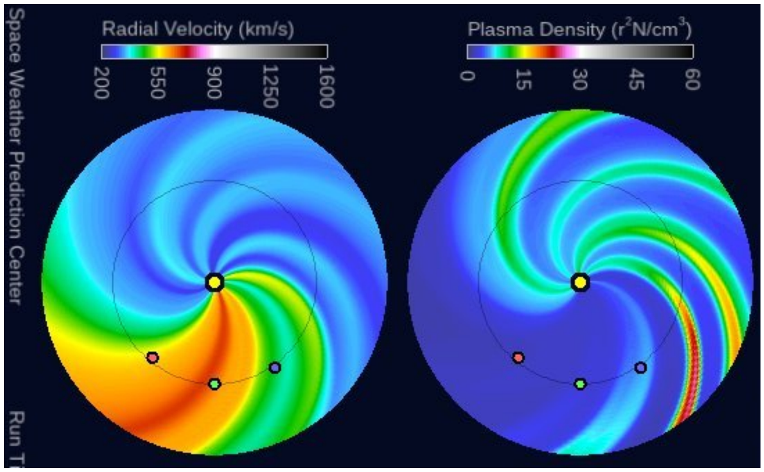

Figure 2:

vi. Within a few planetary radii of the surface, low energy (cold) charged particles present in Earth's equatorial magnetosphere will drift in closed loops around the planet

(b) Describe, using a diagram, the motion of positively charged ions and negatively charged electrons in a uniform, steady background magnetic field aligned with the Z direction, assuming that the both species have a pitch angle $\alpha = 45°$. If the particle then moves into a region where the magnetic field strength is increased by a factor of two, will the parallel energy of the particles increase or decrease to conserve their total energy?

4. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

i. The E layer in the Earth's ionosphere disappears quicker after sunset than does the F layer.
ii. The main reason for ionization in the Earth's ionosphere is the high temperature of the atmosphere at high altitude.
iii. Thanks to electron and ion collisions with the neutral gas molecules, an electric field perpendicular to the magnetic field can give an electric current in the ionosphere.
iv. The aurora is mainly created by ions hitting the atmosphere.
v. The X-ray flux from the Sun increases during a solar flare.
vi. The Kp index shows the number of sunspots in a 3 hour interval.

(b) Figure 2 shows a simulation (from October 18) predicting the solar wind radial velocity (left) and plasma number density (right) in the ecliptic plane at the time you write this exam (October 21, 08:00 UT), viewed from above the Sun's north pole. The green (light, if you are colour blind) circle below the Sun marks the Earth.

i. Many structures in both plots have a spiral shape. Why is that?
ii. According to this forecast, the Earth is right now inside a stream of fast solar wind with low density. What phenomenon on the Sun could give rise to this?
iii. Based on this simulation, what do you think about our chances to see some aurora in Uppsala in the coming week (October 21-27)?
iv. Is there anything the simulation cannot predict which could change the probability of Uppsala aurora in the coming week?

Part B

5. The Solar Orbiter (SolO) spacecraft was launched in February 10, 2020. Figure 3 shows the positions (filled circles) of Earth, Venus, Mercury and SolO at some given times, and the trajectories of these objects up to the same times.

   (a) Did the energy of the SolO orbit around the Sun increase or decrease at the first Venus flyby (in 2020-12-26)? How much (in per cent) did it change? If you find you need a ruler but did not bring one, you can find one in Figure 4. (2 p)

   (b) What do you think the SolO flybys of Venus looked like? Sketch them in a VSO coordinate system (similar to GSE at Earth, i.e. centred in Venus, X pointing to the Sun and Y approximately opposite to the planet's orbital velocity around the Sun) in the ecliptic plane. The sketch should probably extend something like 5-10 Venus radii in each direction. Include arrows showing the direction of flight. You are not supposed to get absolute distances and exact directions correct, just the general characteristics. Describe the reasoning behind your drawing. (3 p)

6. (a) Draw a sketch of the Earth's magnetosphere, viewed from the dusk side (so that the Sun is to the left of the page). Label the following regions, boundaries and features: Earth's magnetic field, magnetopause, bow shock, magnetosheath, magnetosphere, and stream lines of the solar wind. Show the direction of the major electric currents that flow *through* the plane of the page. (2 p)

   (b) Derive an expression for the sub-solar distance from the center of the Earth to the magnetopause, stating any assumptions made. For a solar wind with proton density of 5 cm^{-3} and velocity of 400 km s^{-1}. (2 p)

   (c) Assuming that the magnetosphere expands and contracts similarly in all directions, by how much would you expect the magnetic field strength in the magnetotail to increase if the dynamic pressure of the solar wind increases by a factor of ten? *Hint: one may safely consider the tail as having a circular cross-section.* (2 p)

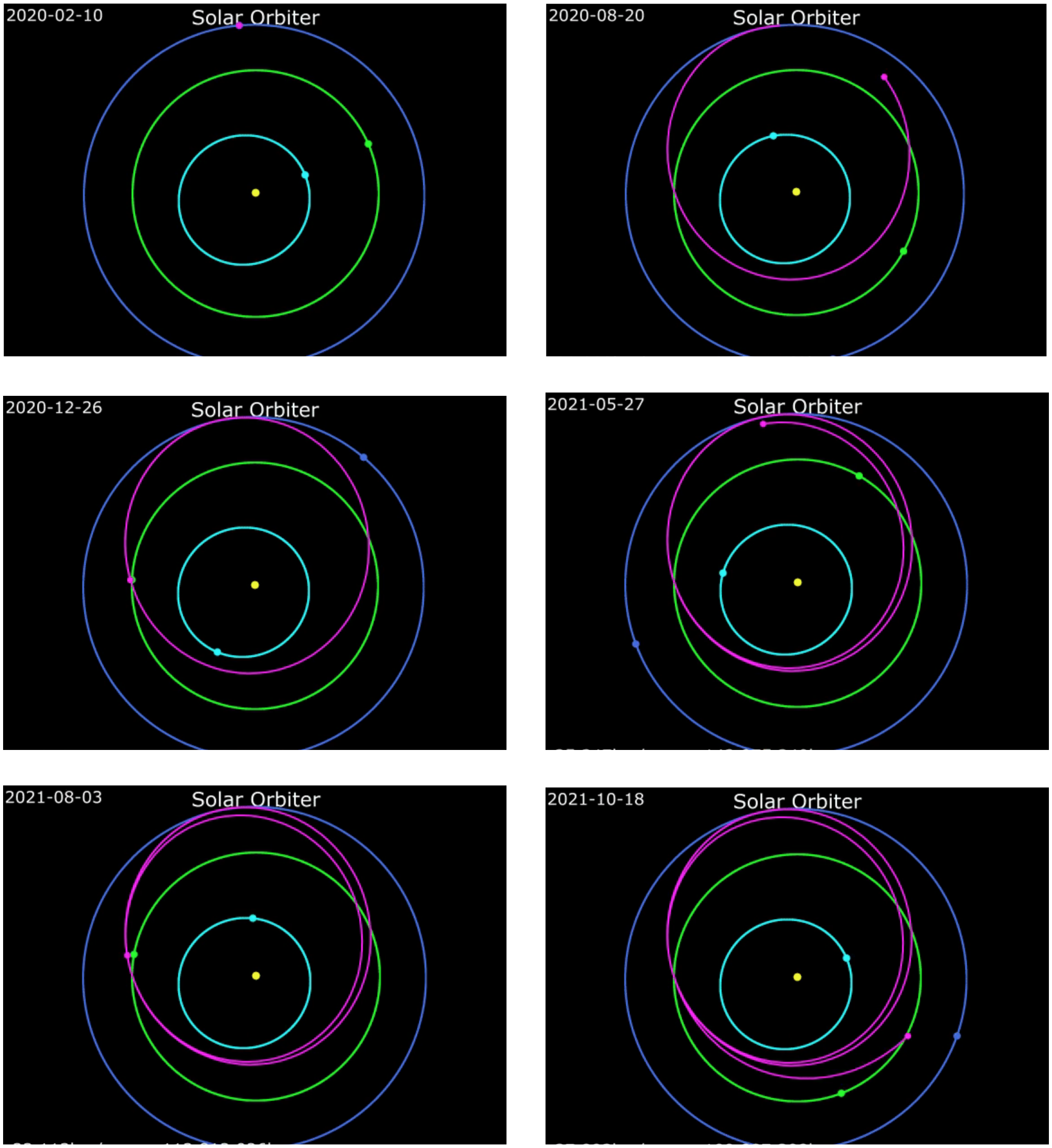
2020-02-10
Solar Orbiter
2020-08-20
Solar Orbiter
2020-12-26
Solar Orbiter
2021-05-27
Solar Orbiter
2021-08-03
Solar Orbiter
2021-10-18
Solar Orbiter

Figure 3:

7. (a) Electrons with pitch angles of 74° are observed by the "FAST" spacecraft at an altitude of 500 km and latitude of 60° North. At what radial distance will these particles cross the equatorial plane, and what will their pitch angle be there? (2 p)

   (b) Consider the motion of individual charged particles in the Earth's (rotating) dipolar magnetic field. Close to the planet, an effective electric field E is present in the frame of the corotating plasma due to the rotation of the Earth's magnetic field. Obtain an expression for the rotating $\boldsymbol{E} \times \boldsymbol{B}$ drift velocity of the particles on the equatorial plane as a function of radial distance. Similarly, starting from the 'general force drift' obtain an expression for the $\nabla \boldsymbol{B}$ drift velocity for particles with perpendicular kinetic energy $W_{\perp}$. Take Earth's equatorial surface field strength to be 31 μT. (2 p)

   (c) Protons with $W_{\perp} = 10\ \mathrm{keV}$ and $W_{||} = 0$ are injected into the magnetosphere at a distance $L = 5$ at local midnight. For these ions, which of the two drift motions calculated above be safely neglected? (1 p)

8. (a) Derive (from the equation of motion of a neutral gas and an assumption of constant gravitational field) an expression showing why the concentrations of neutral molecules decrease approximately exponentially with increasing altitude, and why the concentration of atomic oxygen (O) decreases slower with altitude than the N_2 density, which in turn decreases slower than the concentration of molecular oxygen (O_2). State explicitly all assumptions you make. (2 p)

   (b) Assume the solar intensity (energy per unit area and time) in the extreme ultraviolet (EUV) absorbed at a altitude in an atmosphere is proportional to the number density of the neutral gas and the solar UV intensity itself at that altitude. Using your result from (a), derive how the solar intensity changes with altitude. (2 p)

Lycka till!

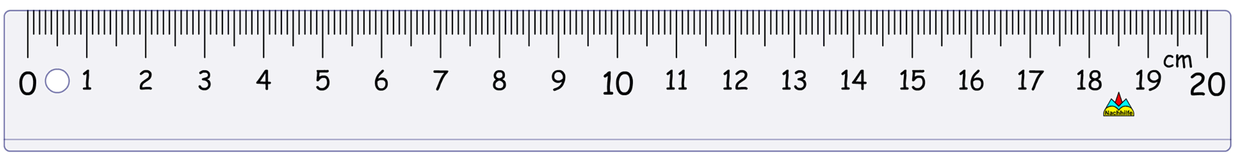

Figure 4: If you did not bring your own ruler, detach this paper and fold it along the upper edge of the image to get a nice working ruler.

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Space Physics Formulas: Complement to Physics Handbook

Charge density and current density from particle species s:

$$\rho = \sum_s q_s n_s, \qquad \mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Galilean transformations:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{B}' = \mathbf{B}$$

Dipole magnetic field:

$$\mathbf{B}(r, \theta) = -B_0 \left(\frac{R_0}{r} \right)^3 \left(2\hat{\mathbf{r}} \cos\theta + \hat{\theta} \sin\theta \right)$$

Dipole field lines:

$$r / \sin^2 \theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m} \frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particle species s:

$$m_s n_s \frac{d\mathbf{v_s}}{dt} = n_s q_s (\mathbf{E} + \mathbf{v_s} \times \mathbf{B}) - \nabla p_s + \text{o.f.}$$

MHD equation of motion:

$$\rho_\mathrm{m} \frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{o.f.} = -\nabla \left(p + \frac{B^2}{2\mu_0} \right) + \frac{1}{\mu_0} (\mathbf{B} \cdot \nabla) \mathbf{B} + \text{o.f.}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Dynamic pressure:

$$p_\mathrm{dyn} = \frac{1}{2} nmv^2$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & -\sigma_\mathrm{H} & 0 \\ \sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_\parallel \end{pmatrix} = \sigma_\mathrm{P} \mathbf{E}_\perp + \sigma_\mathrm{H} \frac{\mathbf{B} \times \mathbf{E}_\perp}{B} + \sigma_\parallel \mathbf{E}_\parallel$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \frac{ne}{B} \left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2 + \nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2 + \nu_\mathrm{e}^2} \right) \\ \sigma_\mathrm{H} &= \frac{ne}{B} \left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2 + \nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2 + \nu_\mathrm{e}^2} \right) \\ \sigma_\parallel &= ne^2 \left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}} \right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_{\rm c} = \omega_{\rm c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_\perp^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu\nabla B$$

Drift motion due to general force **F**:

$$\mathbf{v}_\mathbf{F} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_\perp/v_\parallel$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0}\frac{e^{-r/\lambda_{\rm D}}}{r}$$

Debye length:

$$\lambda_{\rm D} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_{\rm p} = \omega_{\rm p}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_{\rm e}}}$$

Rocket thrust:

$$T = v_{\rm e}\frac{{\rm d}m}{{\rm d}t}$$

Specific impulse:

$$I_{\rm sp} = \frac{\int T\,{\rm d}t}{m_{\rm fuel}g} = v_{\rm e}/g$$

The rocket equation:

$$\Delta v = -gt_{\rm burn} + v_{\rm e}\ln\left(1 + \frac{m_{\rm fuel}}{m_{\rm payload+structure}}\right)$$

Total energy of elliptic orbit of semimajor axis a:

$$E = -\frac{GMm}{2a}$$

Kepler's third law:

$$T^2 \propto a^3$$

Emitted thermal radiation power:

$$P_{\rm e} = \varepsilon\sigma A_{\rm e}T^4$$

Absorbed solar radiation power:

$$P_{\rm a} = \alpha A_{\rm a} I_{\rm rad}$$

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