Hemtentamen (del B) för kursen Rymdfysik (1FA255)
2020-10-23

Uppsala universitet
Institutionen för fysik och astronomi
Avdelningen för astronomi och rymdfysik
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Answers should be provided in Swedish or English.

Time: 08:00 - 13:00

This is part B of the exam. To pass the course, you must pass the oral **part A** or the corresponding examlets. **Part B** must be solved if (and only if) you aim for a higher grade than ”pass” (3 in the 3-4-5 system).

This exam is an individual effort. You must not interact with anybody else during the exam.

This is an open book exam. You can use whatever tools, notes or literature you wish. However, you must motivate or derive relations not found in the enclosed formula sheet, in Physics Handbook, or which are not standard mathematical relations (vector formulae, integrals, etc). Be sure to motivate, describe and/or comment on the steps you take in your calculations, and to define the symbols you use.

Recommended tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet, pencil, paper, ruler, calculator, possibly a protractor.

1. Figure 1 shows the present trajectories of some spacecraft in the inner solar system.
   (a) Why do the trajectories look so weird, apparently not at all Keplerian? How can one from the plot recreate more familiarly looking orbits? By using the indicated positions of Parker Solar Probe (PSP) at the first day of each month, a ruler and some thinking, map these points to a plot where the orbit of this particular spacecraft should look more elliptic. What is the physical relation between your plot and the original? (2 p)
   (b) Based on information only from the figure and its caption, estimate the semimajor axis of PSP by two independent methods. (1 p)
   (c) If the Solar Orbiter spacecraft was a passive piece of matter, with no heating or cooling systems, with equilibrium temperature 60 deg C at Earth orbit, what would be its temperature when closest to the Sun in this time period? Be sure to state all your assumptions. (2 p)

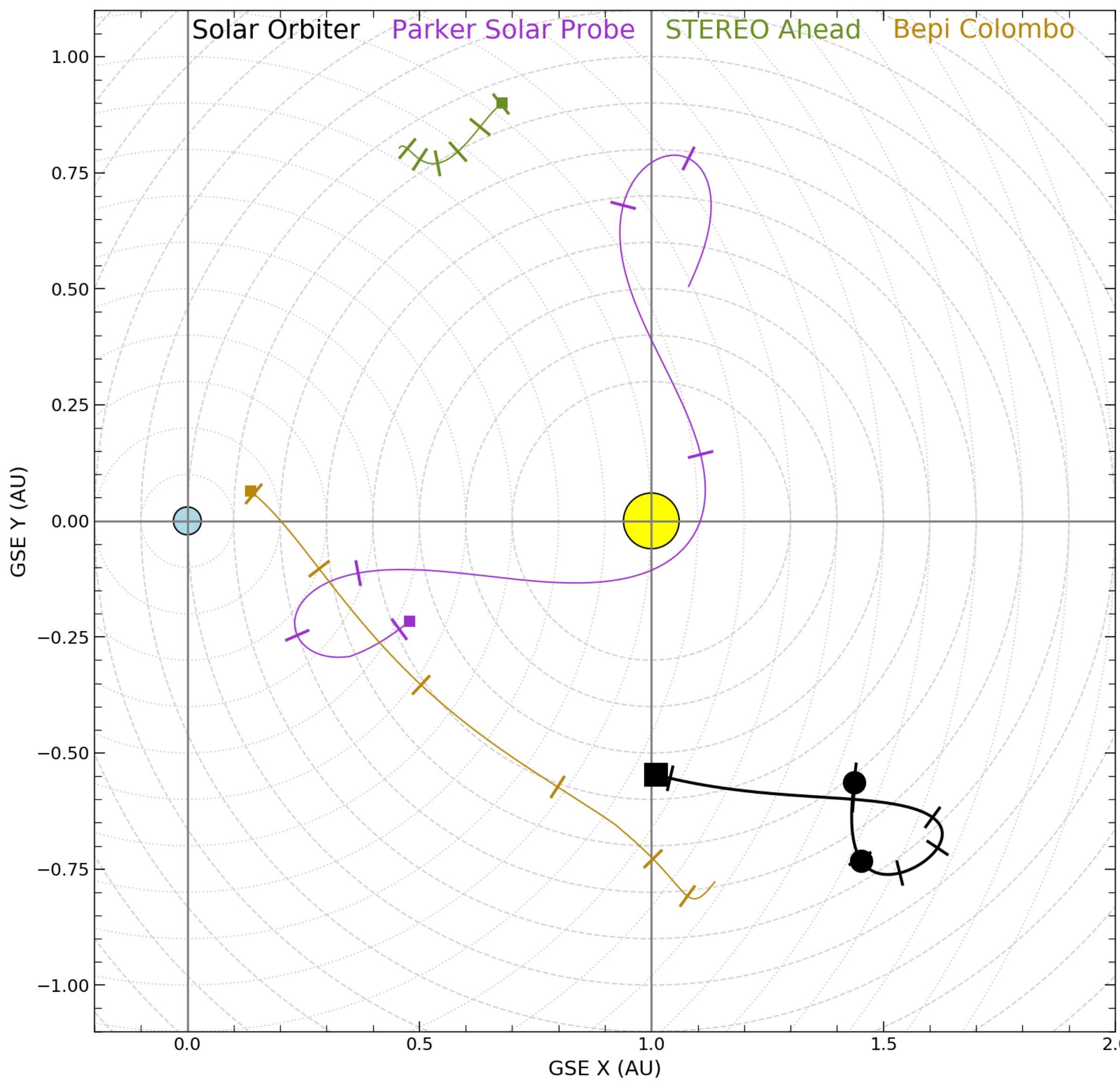

Figure 1: Trajectories of some spacecraft in the inner solar system for the period June 29 – December 29, 2020. The squares at one end of each trajectory shows the spacecraft position in June 29. Tick marks indicate the first day in a month. For Solar Orbiter, the November 1 and December 1 tick marks are instead represented as filled circles. Image credit: ESA

2. Typical parameters in the solar wind at 1 AU can be taken to be $n = 5\text{cm}^{-3}$, $v = 400$ km/s, $B = 5$ nT and Parker spiral angle 135 or 315 deg. Jupiter is about 5 AU from the Sun, has a radius around 70,000 km and a surface magnetic field strength at the equator of approximately 420 μT.

   (a) What IMF strength and Parker spiral angle do you expect at Jupiter orbit? (2 p)

   (b) Assuming Jupiter's magnetosphere is a scaled up version of Earth's (reality is not so simple, but here we do not care), estimate the distance of Jupiter's magnetopause along the line from the planet to the Sun. (2 p)

   (c) In the frame of a planet there usually is a non-zero electric field in the solar wind. Using some suitable assumptions, estimate what potential drop this electric field creates over the magnetospheres of Earth and Jupiter in the terminator plane (the plane through the planet centre perpendicular to the solar direction, i.e. the GSE X-Y plane). (2 p)

3. Consider an electron of energy 1 keV mirroring at an altitude of 5,000 km on a dipolar field line in the Earth's magnetosphere at (magnetic) latitude 65 deg.

   (a) What will be its pitch angle when it crosses the (magnetic) equatorial plane? At what geocentric distance is this crossing? (2 p)

   (b) By some process, an upward directed electric field parallel to the magnetic field happens to form right at the mirror point of this electron. This E-field is very localized and can therefore be seen as a potential jump $\Delta\Phi$ over a very small altitude range around the mirror point. How does this potential jump along $\boldsymbol{B}$ change the pitch angle? How strong (in volts) does $\Delta\Phi$ have to be to enable the electron to reach an altitude of 150 km? (3 p)

4. Figure 2 shows a prediction (based on a WSA-ENLIL simulation run in Thursday morning, Oct 21 at 04:00 UT) for the solar wind and IMF during the time you write this exam, taken from "Space Weather Enthusiast's Dashboard" at NOAA (https://www.swpc.noaa.gov/communities/space-weather-enthusiasts). At the same source you can also find other useful material, though https://www.swpc.noaa.gov/products/real-time-solar-wind is actually a better link for (almost) real time solar wind and IMF data than the link they provide.

   Actual measurements at L1 for the last two months are seen in Figure 3. In each panel, the coloured data points refer to the dates on the horizontal axis while the black dots show data 27 days older.

   (a) To what extent can the 2D colour plots at left and centre in Figure 2 (not the time series) be used to predict the space weather at Earth in the coming weeks? What can and what cannot be predicted? Based on these plots, what is your personal space weather forecast coming two weeks? What kind of events could possibly happen, and what would they be due to? Your answer should probably be somewhere between 10 and 25 lines of text. (2 p)

   (b) Are your ideas consistent with the predicted time series at right in Figure 2? Can they in any useful way be compared to the data in Figure 3? If so, what do you find from that comparison? Your answer should probably be somewhere between 10 and 25 lines of text. (2 p)

Lycka till!

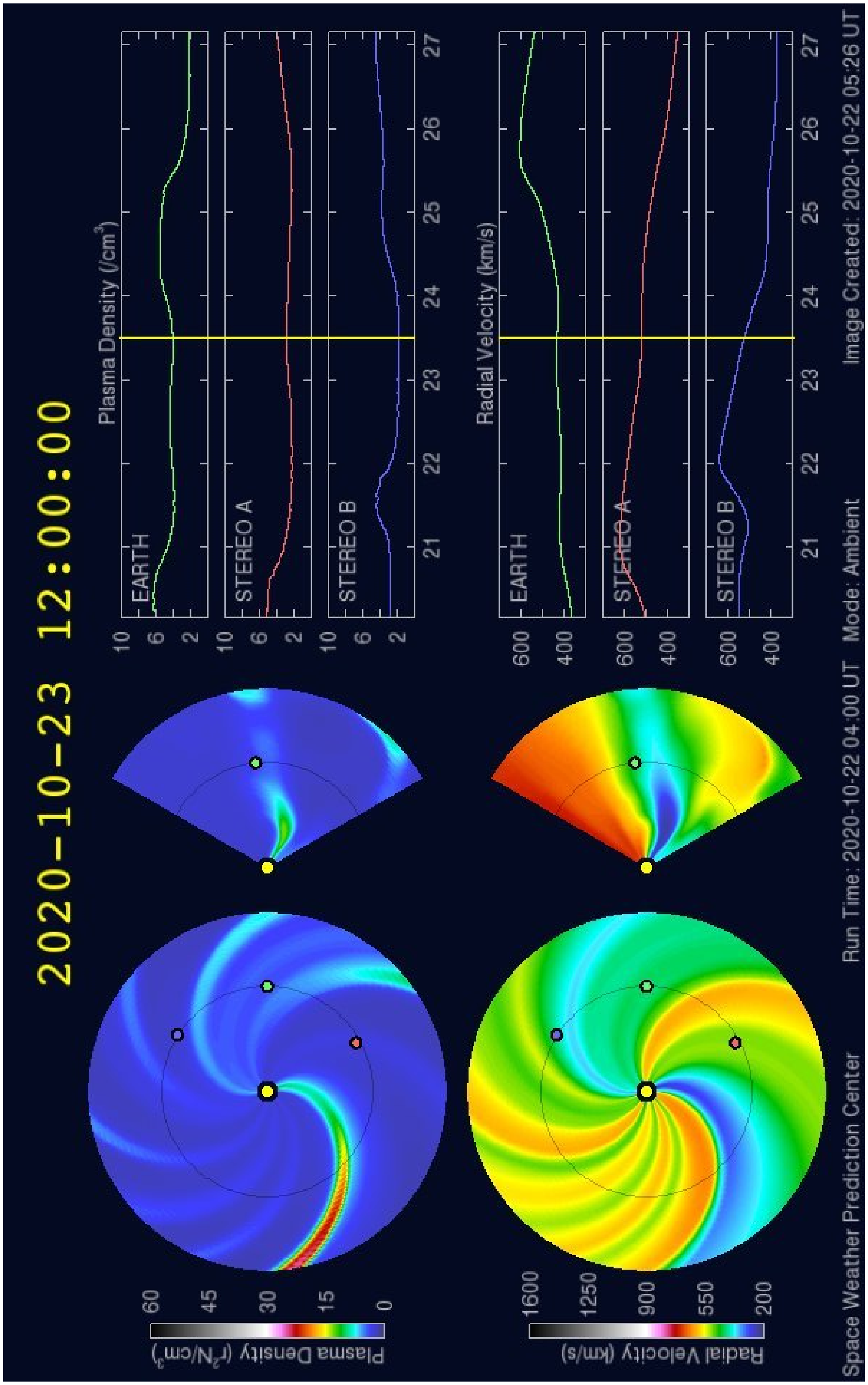

Figure 2: WSA-ENLIL simulation of the solar wind and IMF during the time of this exam. Image credit: NOAA.

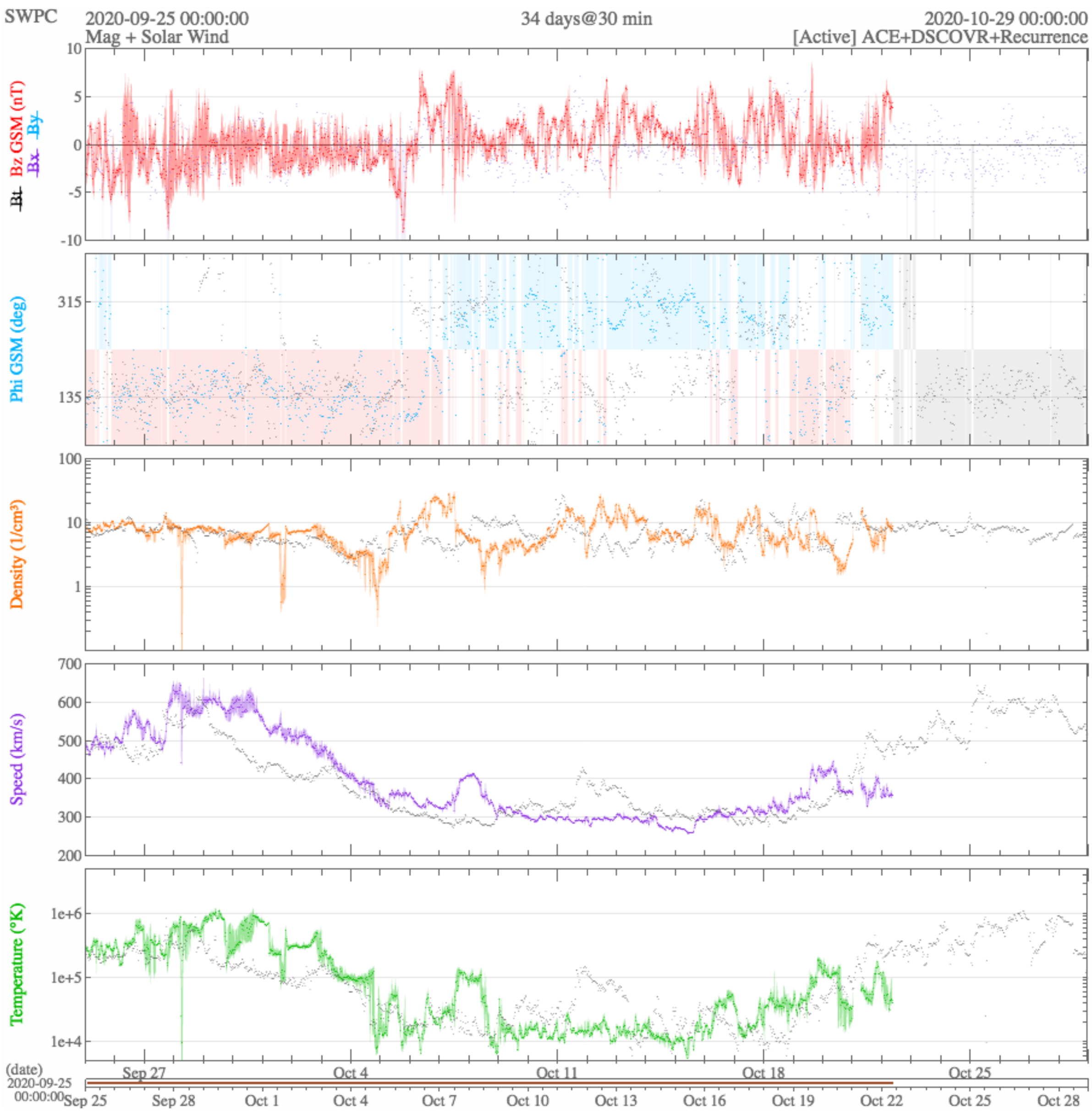

Figure 3: DSCOVR observations at L1 for the last two months. Credit: NOAA.

Space Physics Formulas: Complement to Physics Handbook

Charge density and current density from particle species s:

$$\rho = \sum_s q_s n_s, \qquad \mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Galilean transformations:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{B}' = \mathbf{B}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \mathrm{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_{\mathrm{m}} \frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particle species s:

$$m_s n_s \frac{d\mathbf{v_s}}{dt} = n_s q_s (\mathbf{E} + \mathbf{v_s} \times \mathbf{B}) - \nabla p_s + \text{o.f.}$$

MHD equation of motion:

$$\rho_{\mathrm{m}} \frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{o.f.} = -\nabla\left(p + \frac{B^2}{2\mu_0}\right) + \frac{1}{\mu_0}(\mathbf{B}\cdot\nabla)\,\mathbf{B} + \text{o.f.}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Dynamic pressure:

$$p_{\mathrm{dyn}} = \frac{1}{2} n m v^2$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_{\mathrm{P}} & -\sigma_{\mathrm{H}} & 0 \\ \sigma_{\mathrm{H}} & \sigma_{\mathrm{P}} & 0 \\ 0 & 0 & \sigma_{\parallel} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_{\parallel} \end{pmatrix} = \sigma_{\mathrm{P}} \mathbf{E}_{\perp} + \sigma_{\mathrm{H}} \frac{\mathbf{B} \times \mathbf{E}_{\perp}}{B} + \sigma_{\parallel} \mathbf{E}_{\parallel}$$

Conductivities:

$$\begin{aligned} \sigma_{\mathrm{P}} &= \frac{ne}{B}\left(\frac{\omega_{\mathrm{ci}}\nu_{\mathrm{i}}}{\omega_{\mathrm{ci}}^2+\nu_{\mathrm{i}}^2} + \frac{\omega_{\mathrm{ce}}\nu_{\mathrm{e}}}{\omega_{\mathrm{ce}}^2+\nu_{\mathrm{e}}^2}\right) \\ \sigma_{\mathrm{H}} &= \frac{ne}{B}\left(\frac{\omega_{\mathrm{ci}}^2}{\omega_{\mathrm{ci}}^2+\nu_{\mathrm{i}}^2} - \frac{\omega_{\mathrm{ce}}^2}{\omega_{\mathrm{ce}}^2+\nu_{\mathrm{e}}^2}\right) \\ \sigma_{\parallel} &= ne^2\left(\frac{1}{m_{\mathrm{i}}\nu_{\mathrm{i}}} + \frac{1}{m_{\mathrm{e}}\nu_{\mathrm{e}}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_{\mathrm{c}} = \omega_{\mathrm{c}}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_{\perp}^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu\nabla B$$

Drift motion due to general force **F**:

$$\mathbf{v}_{\mathbf{F}} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_{\perp}/v_{\parallel}$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0}\frac{e^{-r/\lambda_{\mathrm{D}}}}{r}$$

Debye length:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_{\mathrm{p}} = \omega_{\mathrm{p}}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_{\mathrm{e}}}}$$

Rocket thrust:

$$T = v_{\mathrm{e}}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_{\mathrm{sp}} = \frac{\int T\,\mathrm{d}t}{m_{\mathrm{fuel}}g} = v_{\mathrm{e}}/g$$

The rocket equation:

$$\Delta v = -gt_{\mathrm{burn}} + v_{\mathrm{e}}\ln\left(1 + \frac{m_{\mathrm{fuel}}}{m_{\mathrm{payload+structure}}}\right)$$

Total energy of elliptic orbit of semimajor axis a:

$$E = -\frac{GMm}{2a}$$

Kepler's third law:

$$T^2 \propto a^3$$

Emitted thermal radiation power:

$$P_{\mathrm{e}} = \varepsilon\sigma A_{\mathrm{e}}T^4$$

Absorbed solar radiation power:

$$P_{\mathrm{a}} = \alpha A_{\mathrm{a}} I_{\mathrm{rad}}$$

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