Tentamen för kursen Rymdfysik (1FA255)
2019-10-24

Uppsala universitet
Institutionen för fysik och astronomi
Avdelningen för astronomi och rymdfysik
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Answers should be provided in Swedish or English.
Time: 08:00 - 13:00
Allowed tools: Mathematics Handbook (or equivalent), Physics Handbook, enclosed tables and formula sheets, calculator. A bilingual dictionary, for example English-Swedish or English-German, may also be used.
The exam has two parts:

- **Part A** must be satisfactorily solved in order to pass the course. You do not need to solve part A problems corresponding to examlets you have passed during the autumn 2019 course. Part A is graded by pass/fail.
- **Part B** must be solved if you wish to get a higher grade than pass (3 in the Swedish 3-4-5 system, E in ECTS). Grades will depend on the number of points you score on this part.

Part A

1. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.
   i. Rockets are often launched eastward, to take advantage of the Earthâs rotation.
   ii. The mass of a rocket (including fuel) scales exponentially with the velocity increase it provides.
   iii. In an elliptic orbit around Earth, atmospheric friction is higher at perigee than at apogee. Atmospheric friction therefore tends to make the orbit more elliptic, bringing down the perigee while hardly affecting the apogee altitude at all.
   iv. In the geostationary orbit, the gravity from Earth and the Moon cancel, so satellites always stay at the same position.
   v. In a gravity assist manoeuvre, kinetic energy (as seen in the reference frame of the sun) is transferred from a planet to a passing spacecraft.

   (b) Consider a spherical spacecraft with perfect thermal conductivity between all its parts and with all its surface covered by the same material. If its equilibrium temperature is 20°C at 1 AU heliocentric distance, what range of heliocentric distance can it reach if it is designed to survive a temperature range from -20°C to $+80$°C? Internal heat sources may be neglected. (This is a very simplified model, but considerations like these go into real spacecraft design, for evaluating needs for heating and cooling equipment, etc.)

2. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

   i. "Debye-shielding" describes the shielding (removal) of external magnetic fields from a plasma
   ii. In ideal MHD, the magnetic field can be considered to be "frozen in" to the plasma
   iii. In an inertial frame, the solar wind plasma is seen to flow along the Parker spiral
   iv. No current can flow through a plasma if quasi-neutrality is to be maintained
   v. Typically, currents that flow at the magnetopause close (are connected through) the ionosphere

   (b) A stationary flow solution for the solar wind velocity $v(r)$ can be obtained by solving

$$(v^2 - v_T^2)\frac{1}{v}\frac{dv}{dr} = 2\frac{v_T^2}{r} - \frac{GM}{r^2},$$

   where v is the velocity, r is radial distance and GM is the Sun's standard gravitational parameter. What is represented by the term v_T? Sketch a solution appropriate to the solar wind, noting the point where the right hand side of the expression is zero.

3. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

   i. The "guiding center" approximation describes the motion of charged particles a combination of gyromotion about the magnetic field, combined with "drift" motions across the field
   ii. Only a force with a component parallel to the magnetic field will induce a drift motion in a plasma
   iii. The $\mathbf{E} \times \mathbf{B}$ drift does not cause a current to flow
   iv. The magnetic moment of a gyrating particle is an adiabatic invariant, and is conserved only if the field changes slowly relative to the gyroperiod
   v. A particle trapped in the dipolar magnetic field of the Earth, with constant kinetic energy, cannot cross the equator during its motion.
   vi. The gyroradius of a particle must be smaller than the Debye length

   (b) Describe, using a diagram, the motion of positively charged ions and negatively charged electrons in a uniform, steady background magnetic field aligned with the Z direction, where their energies $W_{||} = 0$ and $W_{\perp} \neq 0$. Explain how this leads to the diamagnetic behaviour of a plasma.

4. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

   i. The aurora is mainly created by electrons hitting the atmosphere.
   ii. Magnetospheric substorms in the radiation belts are important sources for corotating interaction regions.
   iii. An magnetosphere will form around any planet with a sufficiently strong magnetic field orbiting a star emitting a stellar wind.
   iv. The main reason for ionization in the Earth's ionosphere is the high temperature of the atmosphere at high altitude.
   v. The E layer in the Earth's ionosphere disappears quickly at night since the ionization source is no longer present and recombination is fast at this altitude where molecular ions dominate.

   (b) Figure 1 shows solar wind observations at L1 (the first Lagrange point of the Sun-Earth system) from October 14-21, 2019. Discuss the data. Were there any particular features seen during this week? Were there any periods you think geomagnetic storms or magnetospheric substorms may have been likely? The space weather forecasts issued around Oct 15 suggested Earth might encounter a corotating interaction region (CIR) around Oct 20. Do you think this happened?

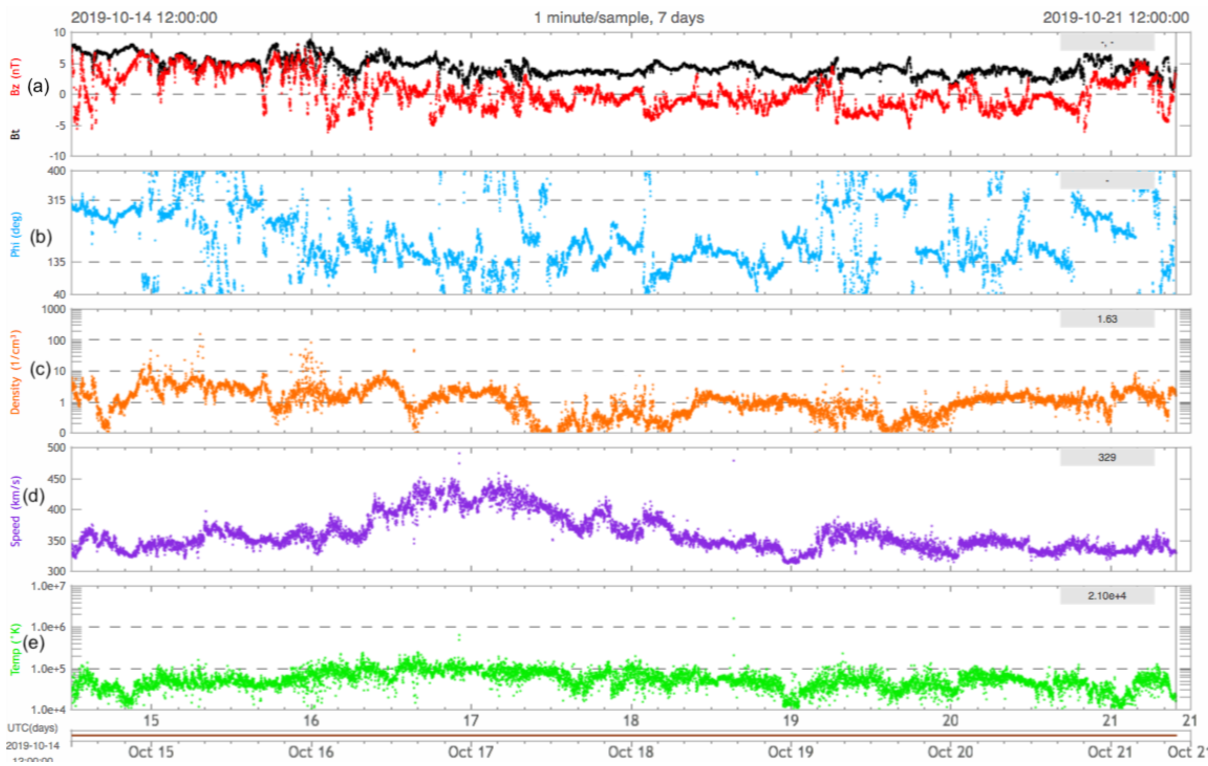

Figure 1: Solar wind data for the week up to October 21, 2019. Panel descriptions: (a) Z (northward, red) component and total strength (black) of the interplanetary magnetic field [nT]. (b) Angle of the magnetic field vector, projected onto the ecliptic plane [°]. The GSE X (sunward) and Y (approximately opposite to Earth's velocity vector around the Sun) axes define 0 and +90°, respectively. (c) Solar wind number density [cm^{-3}]. (d) Solar wind flow speed [km/s]. (e) Ion temperature [K].

Part B

5. In June 2019, ESA selected the Comet Interceptor (CI) mission for launch in December 2027 or later. The goal of CI is to investigate a comet fresh from the Oort cloud (thousands of AU heliocentric distance but still part of the solar system), visiting the inner regions of the solar system for the first time. A comet of this kind is at best discovered a couple of years before it reaches perihelion, but better telescopes will come into service before 2027. To be able to reach such an object, CI will be launched to the L2 Lagrange point of the Earth-Sun system and wait there (possibly for several years) until a suitable comet is discovered. When a target comet has been identified, CI can use its thruster rockets for leaving L2 and set course for the comet. However, because of launch mass limitations, CI will have a very tight fuel budget.

   (a) If the fuel is sufficient for a total velocity change $\Delta v = 2$ km/s, what is the range of heliocentric distances CI can reach? Express your answer as the minimum and maximum possible heliocentric distance, in AU (1 AU = 150,000,000 km, the mean Earth-Sun distance). You may neglect Earth's gravitational influence and assume the journey starts from a circular orbit around the Sun at 1 AU heliocentric distance. The solar mass times Newton's gravitational constant is $GM = 1.33 \cdot 10^{20}$ $\mathrm{m^3/s^2}$. Do not consider gravity assist manoeuvres. (3 p)

   (b) Sketch the CI initial orbit in the solar system (take this to be the Earth orbit) and the trajectories you calculated in (a). If you did not succeed to get an answer in (a), you can still draw what these trajectories should look like and discuss the physics in words to explain your sketch. (1 p)

   (c) Consider the case of the comet trajectory cutting the ecliptic plane at the maximum heliocentric distance CI can reach. How long time in advance must the comet be discovered for us to guarantee that CI is able to meet it? The comet can come from any direction, and you should add 1 month for the planning and decision process from discovery to leaving L2. (2 p)

6. (a) Draw a sketch of the Earth's magnetosphere, viewed from the dusk side (so that the Sun is to the left of the page). Label the following regions, boundaries and features: Earth's magnetic field, magnetopause, bow shock, magnetosheath, magnetosphere, and stream lines of the solar wind. Show the direction of the major electric currents that flow *through* the plane of the page. (2 p)

   (b) Figure 2 shows a bundle of near-horizontal magnetic field lines below the photosphere, in a region you may consider to be otherwise free from magnetic fields. The field is anchored at the ends of the region, i.e. absolutely fixed in the dark shaded region. Write down the MHD equation of motion, and show how, if the system depicted is quasi-stationary, the plasma and magnetic field pressures must balance each other. (1 p)

   (c) Consider the boundary of this bundle of field lines. If the pressures balance across this boundary, what can you infer about the mass density of the plasma inside the bundle compared to outside of it, assuming that the temperature and composition of the plasma is everywhere identical? If the system evolves slowly, what will happen to this bundle of field lines? Show that a force will arise that limits this evolution. (2 p)

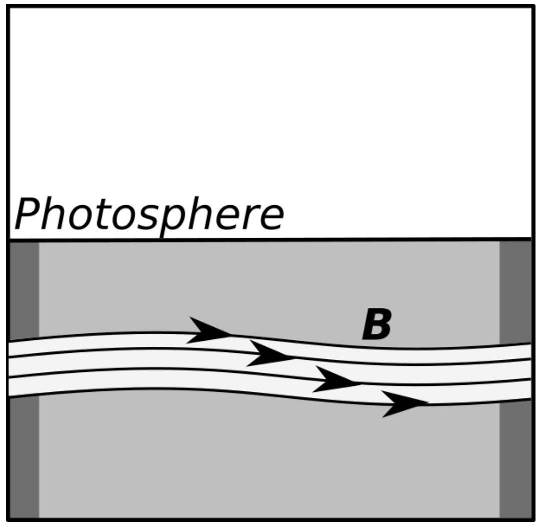

Figure 2: A bundle of magnetic fields below the photosphere.

7. Consider an idealised description of the magnetic field in the Earth's magnetotail, in which the magnetic field in the lobes far from the current sheet $B_L = 20$ nT and reverses sign linearly over a distance of 6000 km.

   (a) Sketch the geometry of the system, and evaluate the current density and the magnetic force density in the current sheet. (2 p)

   (b) The cross-tail current is carried mainly by the ions (assume Oxygen ions, mass 16 amu). If the particle density in the current sheet is 1 cm^{-3}, estimate the velocity the ions must flow with to provide the required current density. (1 p)

   (c) If the total (kinetic and thermal) energy of these ions is 5 keV, suggest whether it is reasonable to expect that the current density could be estimated by a particle detector that operates by measuring the total energy (accurate to 10%) of particles arriving from a given direction. (2 p)

8. (a) Derive (from the equation of motion of a neutral gas and an assumption of constant gravitational field) an expression showing why the concentrations of neutral molecules decrease approximately exponentially with increasing altitude, and why the concentration of atomic oxygen (O) decreases slower with altitude than the N_2 density, which in turn decreases slower than the concentration of molecular oxygen (O_2). State explicitly all assumptions you make. (2 p)

   (b) A typical electron density profile for the Earth's ionosphere, i.e. the electron number density $n(h)$ as a function of height h above ground, typically has $n(0) = n(\infty) = 0$ and a maximum at an altitude h of a few hundred kilometers. What are the physical reasons for why the ionosphere looks like this? (2 p)

Lycka till!

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Galilean transformations:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{B}' = \mathbf{B}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_{\mathrm{c}} = \omega_{\mathrm{c}}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_{\perp}^{2}/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_{B} = -\mu\nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v}_{\mathbf{F}} = \frac{\mathbf{F}\times\mathbf{B}}{qB^{2}}$$

Pitch angle:

$$\tan\alpha = v_{\perp}/v_{\parallel}$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_{0}}\frac{e^{-r/\lambda_{\mathrm{D}}}}{r}$$

Debye length:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\epsilon_{0}KT}{ne^{2}}}$$

Plasma frequency:

$$f_{\mathrm{p}} = \omega_{\mathrm{p}}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^{2}}{\epsilon_{0}m_{\mathrm{e}}}}$$

Rocket thrust:

$$T = v_{\mathrm{e}}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_{\mathrm{sp}} = \frac{\int T\,\mathrm{d}t}{m_{\mathrm{fuel}}g} = v_{\mathrm{e}}/g$$

The rocket equation:

$$\Delta v = -gt_{\mathrm{burn}} + v_{\mathrm{e}}\ln\left(1+\frac{m_{\mathrm{fuel}}}{m_{\mathrm{payload+structure}}}\right)$$

Emitted thermal radiation power:

$$P_{\mathrm{e}} = \varepsilon\sigma A_{\mathrm{e}}T^{4}$$

Absorbed solar radiation power:

$$P_{\mathrm{a}} = \alpha A_{\mathrm{a}}I_{\mathrm{rad}}$$