Tentamen för kursen Rymdfysik (1FA255)
2017-12-20

Uppsala universitet
Institutionen för fysik och astronomi
Avdelningen för astronomi och rymdfysik
Anders Eriksson

Answers should be provided in Swedish or English.

Time: 08:00 - 13:00

Allowed tools: Mathematics Handbook (or equivalent), Physics Handbook, enclosed tables and formula sheets, calculator. A bilingual dictionary, for example English-Swedish or English-German, may also be used.

The exam has two parts:

- **Part A** must be satisfactorily solved in order to pass the course. This part is only graded by pass/fail.
- **Part B** must be solved if you wish to get a higher grade than pass (3 in the Swedish 3-4-5 system, E in ECTS). Grades will depend on the number of points you score on this part.

Part A

1. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

    i. To increase the eccentricity of a satellite orbit, one can use a thruster at perigee so that the applied force is parallel to the velocity of the satellite.

    ii. The main advantage of a multistage rocket is that the air friction can be minimized at any altitude.

    iii. The first Lagrange point L1 is particularly useful for spacecraft monitoring the sun and the solar wind as it always stays above the same point on the Earth's surface.

    iv. Polar orbits are useful for global survey satellites since they can cover all parts of Earth.

    v. In an elliptic orbit around Earth, atmospheric friction is highest at perigee. Atmospheric friction therefore tends to make such an orbit more circular (less eccentric).

    (b) The sensor of our instrument (a kind of space weather station known as a Langmuir probe) on the Cassini spacecraft, orbiting Saturn (heliocentric distance 9.54 AU) until a few months ago, was a sphere ($r = 25$ mm) covered with titanium nitride, with absorption coeffiecient 0.47 and emission coefficient 0.10. Neglecting heat transport from the spacecraft, what was the equilibrium temperature of the sensor?

2. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

    i. The magnetic field in the frame of reference of the plasma is close to zero (for processes on sufficiently large scales in time and space).

    ii. If the electric field in a plasma, as measured in its rest frame, is everywhere zero, two particles initially connected by a magnetic field line always will be so.

iii. At 1 AU, the solar wind energy density is dominated the thermal contribution $\sim nKT$ (rather than the magnetic energy density $B^2/(2\mu_0)$ or bulk kinetic energy density $mnv^2/2$).

iv. The magnetic pressure arises because of the random motion of magnetized particles.

v. The typical direction of the interplanetary magnetic field is about 45 degrees from the Sun.

(b) The attached figure shows 24 hours of solar wind data from the DSCOVR spacecraft acquired on April 30, 2017 near the first Sun-Earth Lagrange point. Magnetic field data are given in the GSE coordinate system, in which $\hat{\mathbf{x}}$ points to the sun. From top to bottom, the plots show B_x, B_y, B_z (all in nT) and the solar wind speed in km/s.

With suitable assumptions, calculate all three GSE components of the electric field which would be measured by an electric field instrument on DSCOVR at time 12:00.

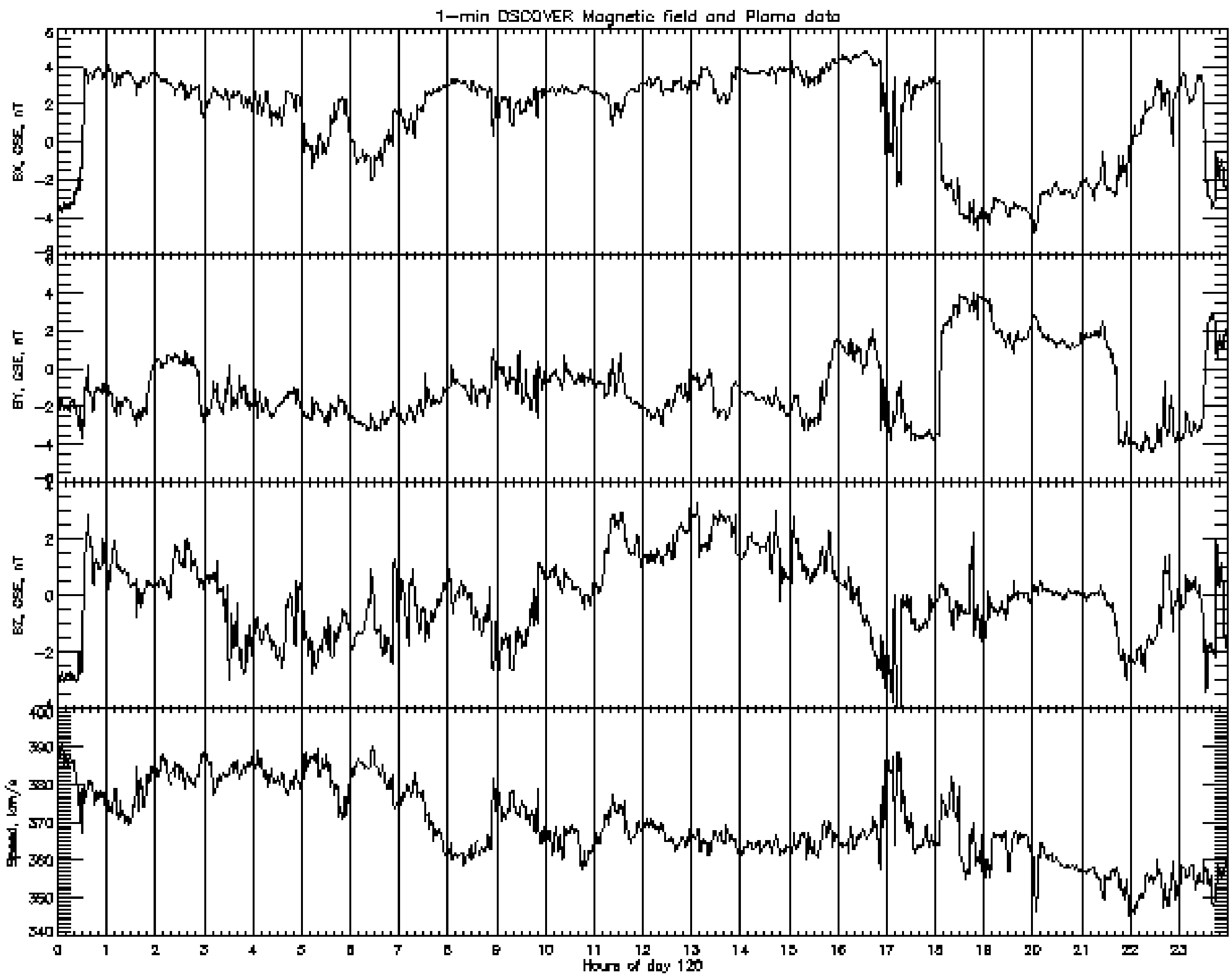


Figure 1: Magnetic field and solar wind data from DSCOVR.

3. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

i. The magnetic moment due to the gyration of a charged particle in a magnetic field is an adiabatic invariant if the magnetic field changes very little over a gyroperiod and gyroradius.

ii. In a homogeneous magnetic field, all particles of the same species, for example electrons, have the same gyroradius.

iii. An electric field perpendicular to the magnetic field (both fields homogeneous and static) causes an electric current to flow in a collisionless plasma.
iv. In general, the drift period is longer than the bounce period, for particles moving in dipolar fields.
v. In a homogeneous magnetic field, all particles of the same species, for example electrons, have the same gyroradius.

(b) An electron has pitch angle 60° in the equatorial plane at geocentric distance 3 RE. Can it reach the a point at geocentric distance 2 RE on the same field line?

4. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

i. An magnetosphere will form around any planet with an atmosphere if it orbits a star emitting ionizing radiation.
ii. The aurora is mainly due to keV electrons hitting the upper atmosphere.
iii. Magnetospheric substorms are primarily associated with the nightside part of the magnetosphere, while geomagnetic storms are global phenomena also affecting the dayside magnetosphere.
iv. Dissociative recombination is an important plasma loss mechanism in the Earth's ionosphere.
v. The F layer in the Earth's ionosphere disappears quickly at night since the ionization source is no longer present and the recombination is fast at this altitude.
vi. Due to collisions, an electric field perpendicular to the magnetic field can drive a current in the ionosphere but not in the magnetosphere.

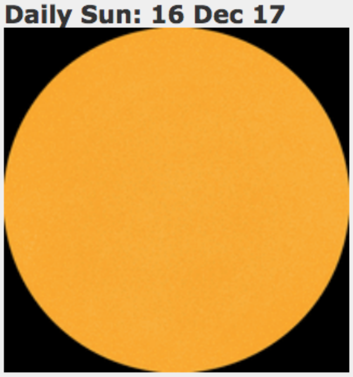
(b) Figure 2 shows the space weather status and forecast from spaceweather.com as of 22:00 UT in Dec 16, 2017. Figure 3 shows solar wind data acquired upstream of the Earth at the first Lagrange point for Dec 16–18. Which aspects of the forecast can you actually compare to the data for Dec 16–18 in Figure 3? Is there something you cannot directly compare? How well do you think the forecast did, as far as can be seen in Figure 3? When do you think the chances to see any aurora in Uppsala in this period were best? Your answer should probably be between ten lines and one page in length.

Current Conditions

Solar wind
speed: **367.1** km/sec
density: **20.0** protons/cm^3
more data: ACE, DSCOVR
Updated: Today at 2351 UT

X-ray Solar Flares
6-hr max: **A4** **2032 UT Dec16**
24-hr: **A8** **0618 UT Dec16**
explanation | more data
Updated: Today at: 2300 UT

Daily Sun: 16 Dec 17



The sun is blank--no sunspots. Credit: SDO/HMI

Sunspot number: 0
What is the sunspot number?
Updated 16 Dec 2017

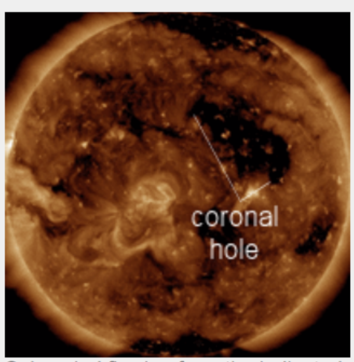
Spotless Days
Current Stretch: 3 days
2017 total: 97 days (27%)
2016 total: 32 days (9%)
2015 total: 0 days (0%)
2014 total: 1 day (<1%)
2013 total: 0 days (0%)
2012 total: 0 days (0%)
2011 total: 2 days (<1%)
2010 total: 51 days (14%)
2009 total: 260 days (71%)
Updated 16 Dec 2017

The Radio Sun
10.7 cm flux: **72** sfu
explanation | more data
Updated 15 Dec 2017

Planetary K-index
Now: **Kp= 1** quiet
24-hr max: **Kp= 1** quiet
explanation | more data

Interplanetary Mag. Field
B_{total}: **8.2** nT
B_z: **-1.7** nT **south**
more data: ACE, DSCOVR
Updated: Today at 2351 UT

Coronal Holes: 16 Dec 17



Solar wind flowing from the indicated coronal hole could reach Earth on Dec. 17-18. Credit: SDO/AIA

SPACE WEATHER
NOAA Forecasts

Updated at: 2017 Dec 16 2200 UTC

FLARE	0-24 hr	24-48 hr
CLASS M	01 %	01 %
CLASS X	01 %	01 %

Geomagnetic Storms:
Probabilities for significant disturbances in Earth's magnetic field are given for three activity levels: active, minor storm, severe storm

Updated at: 2017 Dec 16 2200 UTC

Mid-latitudes

	0-24 hr	24-48 hr
ACTIVE	25 %	40 %
MINOR	10 %	25 %
SEVERE	01 %	05 %

High latitudes

	0-24 hr	24-48 hr
ACTIVE	15 %	10 %
MINOR	30 %	25 %
SEVERE	35 %	55 %

Figure 2:

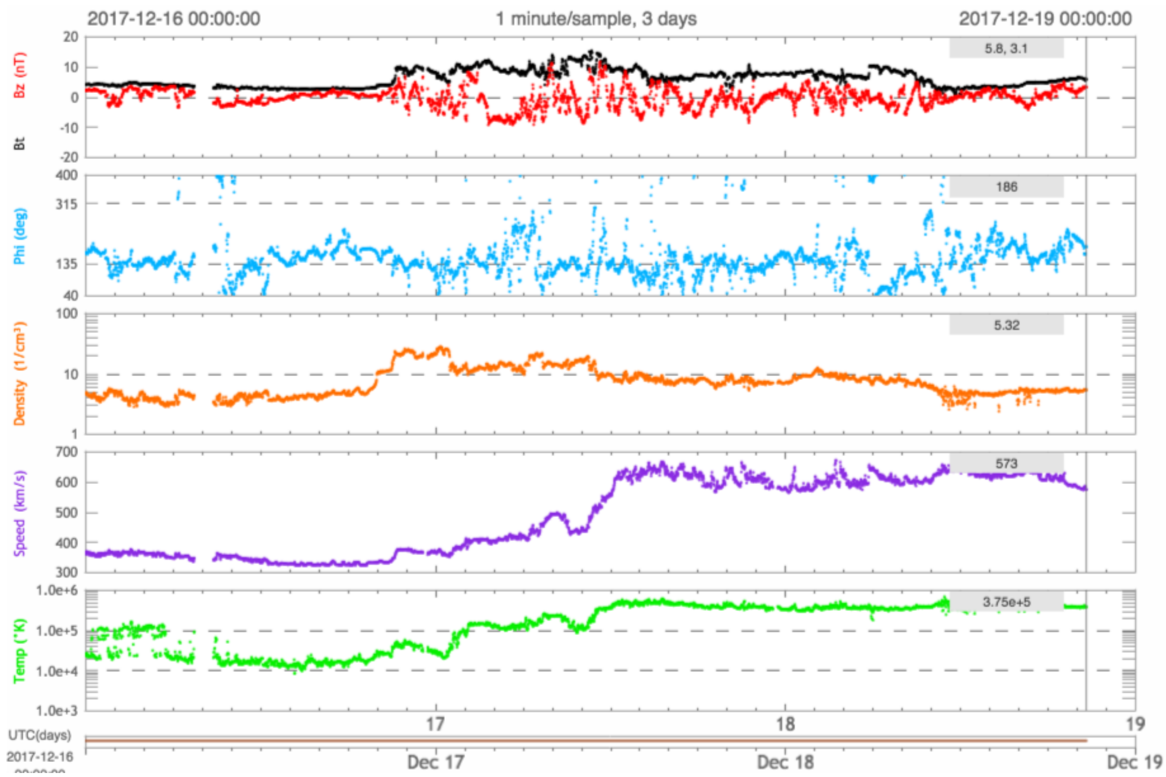

2017-12-16 00:00:00
1 minute/sample, 3 days
2017-12-19 00:00:00
5.8, 3.1
Bz (nT)
Bt
186
Phi (deg)
5.32
Density (1/cm³)
573
Speed (km/s)
3.75e+5
Temp (°K)
UTC(days)
2017-12-16
17
18
19
Dec 17
Dec 18
Dec 19

Figure 3:

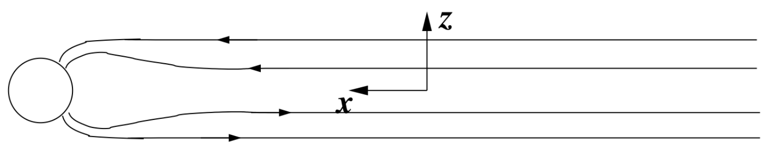


Figure 4: Idealized geometry of the geomagnetic tail.

Part B

5. Two satellites known as the Radiation Belt Storm Probes (RBSP), after launch renamed the Van Allen probes, were launched from Cape Canaveral Air Station (latitude 28°N, longitude 80.6°W) in August 30, 2012. As the name suggests, their primary purpose is detailed investigation of the Earth's radiation belts.

   (a) If the launch would have been as efficient as possible, what inclination would the RBSP orbit have? Motivate in words why this is so. (1 p)

   (b) Actually, the RBSP inclination is about 10°, and the orbit is elliptical with perigee and apogee altitudes of about 600 km and 30,000 km, respectively. Estimate the highest and lowest magnetic field strengths you expect the magnetometers on board RBSP does observe during an orbit. (2 p)

   (c) The two satellites were launched by an Atlas V rocket, whose Centaur upper stage with the satellites onboard first went into a circular parking orbit at 600 km altitude. A final boost then brought the satellites into their final orbit. If each satellite weighs 500 kg, how much work must be done on it to bring it from the parking orbit into the final orbit? (2 p)

   (d) Is there any particular design issue you would expect has been more problematic for RBSP than for many other scientific satellites orbiting the Earth? Motivate your answer. (1 p)

   As usual, the geomagnetic field may be approximated as a dipole field with strength 30 μT on the ground at the equator, with the dipole axis aligned to the Earth spin axis.

6. (a) Show that the kinetic energy of a charged particle moving in a magnetic field, which is constant in time but may vary in space, is constant. (2 p)

   (b) Consider an oxygen ion (O^+) with a kinetic energy of 10 keV and no velocity along the magnetic field, moving in the equatorial plane at a distance of 3 R_E from the center of the Earth. Calculate the two characteristic frequencies defined for this particle (the third one is undefined because of the particle's zero velocity along the magnetic field). Also calculate the gyroradius of the ion. Is it reasonable to describe the motion of the ion as the superposition of two periodic motions as done above?(4 p)

   The geomagnetic field may be taken to be a dipole field with strength 30 μT on the ground at the equator.

7. Consider the following model of the magnetic field in the central part of the geomagnetic tail:

$$\mathbf{B}(\mathbf{r}) = \begin{cases} -B_0\hat{\mathbf{x}} & , \quad z < -a \\ B_0\,\hat{\mathbf{x}}\,\frac{3\,a^2\,z - z^3}{2\,a^3} & , \quad -a \le z \le a \\ B_0\hat{\mathbf{x}} & , \quad z > a \end{cases}$$

   where $B_0 = 1$ nT, $a = 2000$ km and the coordinates are defined as in Figure 4.

   (a) Calculate the current density $\mathbf{j}(\mathbf{r})$ and the magnetic force density $\mathbf{j}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})$ (magnitudes and directions as functions of position). Also calculate their numerical values at $z = 0$. (3 p)

   (b) Now consider what happens if an instability appears in the region $-a < x < a$, $-10\,a < y < 10\,a$, $-a < z < a$ so that the resistivity in this region includes drastically. When the currents cannot flow through this region as before, where will they close now? How much magnetic energy is stored in this volume? What happens if this is released during a few minutes? (2 p)

8. GI 581g is an exoplanet reported in 2010 to orbit the star Gliese 581 and speculated to have conditions favorable for life. However, even its existence could not be confirmed by other observations. We thus have a lot of freedom to speculate freely, so let us assume it has an atmosphere which at high altitudes mainly consists of molecular oxygen O_2. The main components of the ionosphere could then be oxygen ions O_2^+ and electrons, and we assume a peak density 10^6 cm^{-3}. Assume that the loss process is dissociative recombination $O_2^+ + e \rightarrow O + O$ with reaction constant $\alpha = 6 \cdot 10^{-14}$ m^3/s. Estimate the typical lifetime of an oxygen ion O_2^+ ion. The rotation of the planet gives a day of about 26 hours. Will the ionosphere of GI 581g disappear at night or will it persist? (3 p)

Lycka till!

Space Physics Formulas: Complement to Physics Handbook

Charge density and current density from particle species s:

$$\rho = \sum_s q_s n_s, \qquad \mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Galilean transformations:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{B}' = \mathbf{B}$$

Dipole magnetic field:

$$\mathbf{B}(r, \theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}} \cos\theta + \hat{\theta} \sin\theta\right)$$

Dipole field lines:

$$r / \sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_{\mathrm{m}} \frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particle species s:

$$m_s n_s \frac{d\mathbf{v_s}}{dt} = n_s q_s (\mathbf{E} + \mathbf{v_s} \times \mathbf{B}) - \nabla p_s + \text{o.f.}$$

MHD equation of motion:

$$\rho_{\mathrm{m}} \frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{o.f.} = -\nabla \left(p + \frac{B^2}{2\mu_0}\right) + \frac{1}{\mu_0} (\mathbf{B} \cdot \nabla) \mathbf{B} + \text{o.f.}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Dynamic pressure:

$$p_{\mathrm{dyn}} = \frac{1}{2} n m v^2$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_{\mathrm{P}} & -\sigma_{\mathrm{H}} & 0 \\ \sigma_{\mathrm{H}} & \sigma_{\mathrm{P}} & 0 \\ 0 & 0 & \sigma_{\parallel} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_{\parallel} \end{pmatrix} = \sigma_{\mathrm{P}} \mathbf{E}_{\perp} + \sigma_{\mathrm{H}} \frac{\mathbf{B} \times \mathbf{E}_{\perp}}{B} + \sigma_{\parallel} \mathbf{E}_{\parallel}$$

Conductivities:

$$\begin{aligned} \sigma_{\mathrm{P}} &= \tfrac{ne}{B} \left(\tfrac{\omega_{\mathrm{ci}} \nu_{\mathrm{i}}}{\omega_{\mathrm{ci}}^2 + \nu_{\mathrm{i}}^2} + \tfrac{\omega_{\mathrm{ce}} \nu_{\mathrm{e}}}{\omega_{\mathrm{ce}}^2 + \nu_{\mathrm{e}}^2} \right) \\ \sigma_{\mathrm{H}} &= \tfrac{ne}{B} \left(\tfrac{\omega_{\mathrm{ci}}^2}{\omega_{\mathrm{ci}}^2 + \nu_{\mathrm{i}}^2} - \tfrac{\omega_{\mathrm{ce}}^2}{\omega_{\mathrm{ce}}^2 + \nu_{\mathrm{e}}^2} \right) \\ \sigma_{\parallel} &= ne^2 \left(\tfrac{1}{m_{\mathrm{i}} \nu_{\mathrm{i}}} + \tfrac{1}{m_{\mathrm{e}} \nu_{\mathrm{e}}} \right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_{\mathrm{c}} = \omega_{\mathrm{c}}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_{\perp}^{2}/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_{B} = -\mu\nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v}_{\mathbf{F}} = \frac{\mathbf{F}\times\mathbf{B}}{qB^{2}}$$

Pitch angle:

$$\tan\alpha = v_{\perp}/v_{\parallel}$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_{0}}\frac{e^{-r/\lambda_{\mathrm{D}}}}{r}$$

Debye length:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\epsilon_{0}KT}{ne^{2}}}$$

Plasma frequency:

$$f_{\mathrm{p}} = \omega_{\mathrm{p}}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^{2}}{\epsilon_{0}m_{\mathrm{e}}}}$$

Rocket thrust:

$$T = v_{\mathrm{e}}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_{\mathrm{sp}} = \frac{\int T\,\mathrm{d}t}{m_{\mathrm{fuel}}g} = v_{\mathrm{e}}/g$$

The rocket equation:

$$\Delta v = -gt_{\mathrm{burn}} + v_{\mathrm{e}}\ln\left(1 + \frac{m_{\mathrm{fuel}}}{m_{\mathrm{payload+structure}}}\right)$$

Total energy of elliptic orbit of semimajor axis a:

$$E = -\frac{GMm}{2a}$$

Emitted thermal radiation power:

$$P_{\mathrm{e}} = \varepsilon\sigma A_{\mathrm{e}}T^{4}$$

Absorbed solar radiation power:

$$P_{\mathrm{a}} = \alpha A_{\mathrm{a}}I_{\mathrm{rad}}$$

aie171019