Tentamen för kursen Rymdfysik (1FA255)
2014-10-24

Uppsala universitet
Institutionen för fysik och astronomi
Avdelningen för astronomi och rymdfysik
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Answers should be provided in Swedish or English.
Time: 8:00 - 13:00
Allowed tools: Mathematics Handbook (or equivalent), Physics Handbook, enclosed tables and formula sheets, calculator. A bilingual dictionary, for example English-Swedish or English-German, may also be used.
The exam has two parts:

- **Part A** must be satisfactorily solved in order to pass the course. However, if you have passed some of the examlets 1-4, you do not have to solve the corresponding problem in Part A. Which of the examlets you have passed was stated in an e-mail to you in October 23 and the information is also available on a list in the exam hall. This part is only graded by pass/fail.
- **Part B** must be solved if you wish to get a higher grade than pass (3 in the Swedish 3-4-5 system, E in ECTS). Grades will depend on the number of points you score on this part.

Part A

1. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.
   i. Specific impulse I_{sp} is a measure of the rocket performance. Typical values are 100-500 s. The higher the value of I_{sp}, the less propellant must be used for a given thrust.
   ii. The distance r between the center of a central body and a satellite in a circular orbit around it, depends on the satellite velocity v as $r \propto \frac{1}{v^2}$
   iii. Kepler's 2nd law tells us that a body (e.g. a satellite, moon, or planet) in an elliptical orbit around a central body (e.g. moon, planet, or star) moves slower when it is closer to the central body than when it is further away.
   iv. The eccentricity e of a circular orbit is 0.
   v. Polar orbits are useful for global survey satellites since they can cover all parts of Earth.
   vi. If we fire our rocket at apogee then the speed and energy of the satellite increases so it will go into a larger orbit with a larger eccentricity.

   (b) Calculate the orbital period of a satellite in a circular orbit around Earth at distance 2 R_E.

2. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.
   i. Debye shielding means that the magnetic flux only can be transported along an electric field.
   ii. As seen in an inertial frame, the solar wind flows radially away from the sun.

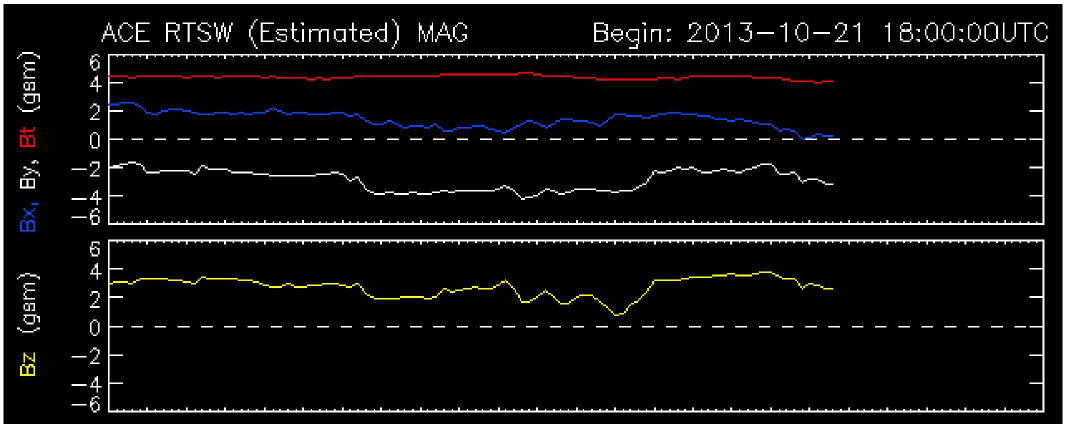

Figure 1: ACE magnetic field data. The xyz components are given in the GSE system. Bt is the magnitude of the magnetic field. The plot covers 2 hours, with data available for about 95 minutes.

iii. If the magnetic field is frozen into the plasma, two plasma elements which at one time are on the same magnetic field lines will always be so.

iv. If the magnetic field varies on time scales shorter than the gyrofrequency, it need not be frozen into the plasma.

v. If the magnetic field varies on spatial scales shorter than the gyroradius, it need not be frozen into the plasma.

(b) Figure 1 shows 95 minutes of magnetic field measurements from the ACE spacecraft in the solar wind close to the Earth but well outside the bow shock. Discuss the magnetic field observation in relation to the Parker spiral, and estimate the electric field (magntitude and direction) an electric field instrument on ACE would likely have measured (if it existed – there is no electric field instrument on ACE).

3. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can if you wish add comments if you feel the need to do so. For question 1, please give the answer in the form d = plasma sphere, e = radiation belts, etc.

i. In Figure 2 of the magnetosphere the regions marked d and e give the position of the plasmasphere and the radiation belts. What are the names of the regions and features marked a, b, c, and f?

ii. The orbital magnetic moment μ of an electron moving in a magnetic field is conserved if the field varies a lot during an electron gyroperiod or inside an electron gyroradius.

iii. The planetary magnetic field of Earth is in the direction pointing from the geographical south towards the geographical north.

iv. The bow shock is characterized by an abrupt transition in flow speed, density, temperature, and magnetic field strength.

v. A particle moving in a dipole field can be trapped between mirror points if its pitch angle is 90° well above the ionosphere.

vi. A magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}$ and an electric field $\mathbf{E} = E_0\hat{\mathbf{y}}$ will generate a current in the $\hat{\mathbf{x}}$ direction in a collisionless plasma.

(b) Estimate the drift speed due to the ∇B-drift for a hydrogen ion at 3 R_{E} with kinetic energy of 1 eV and pitch angle $\alpha = 90°$ in the equator plane. The mean radius of Earth is 6371 km, the surface magnetic field strength at the equator 30 μT, and 1 eV = $1.6 \cdot 10^{-19}$J.

4. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

i. An ionosphere will form around any planet with an atmosphere and some source of ionizing radiation, for example UV radiation from the star it is orbiting.

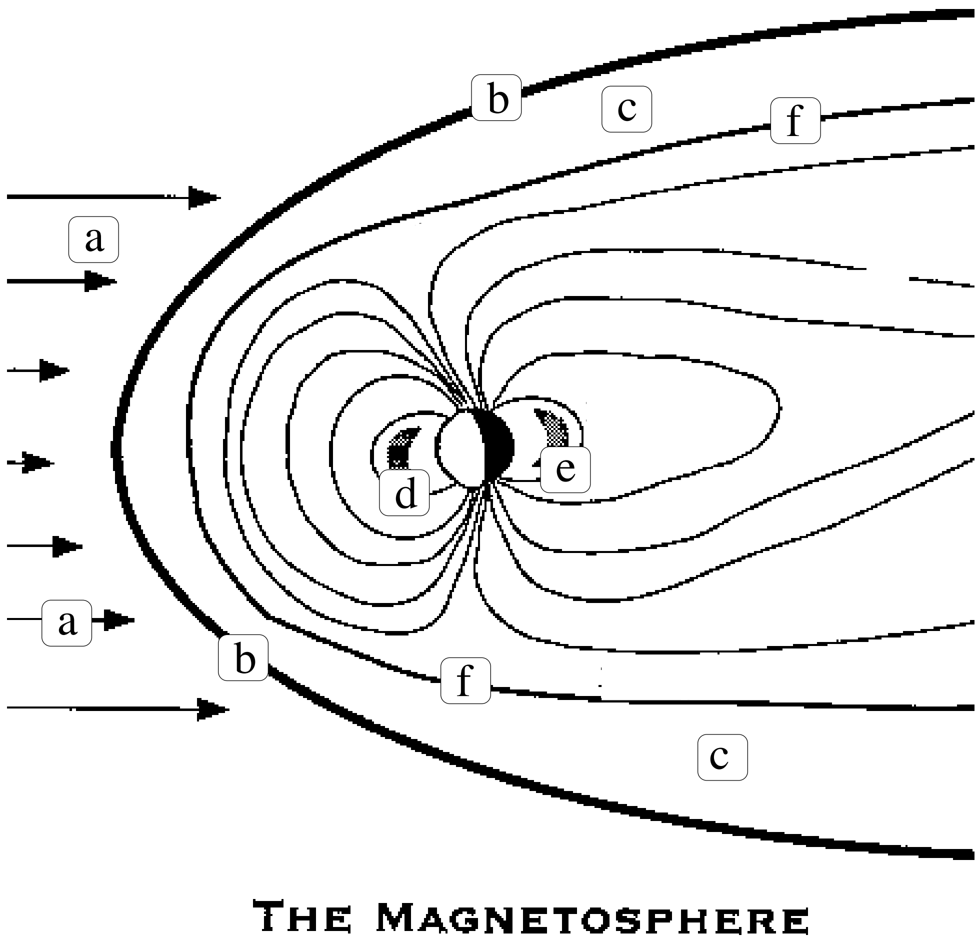

Figure 2:

ii. Magnetospheric substorms are primarily associated with the nightside part of the magnetosphere, while geomagnetic storms are global phenomena also affecting the dayside magnetosphere.
iii. The aurora is mainly created by ions accelerated in regions where field aligned currents flow.
iv. Geomagnetic storms are triggered by solar wind disturbances, often due to coronal mass ejections or solar flares.
v. Recombination is very fast as positive and negative particles attract, leading to almost complete loss of all plasma in the ionosphere during nighttime.

(b) Which component of the interplanetary magnetic field is usually most interesting from a space weather point of view? Why?

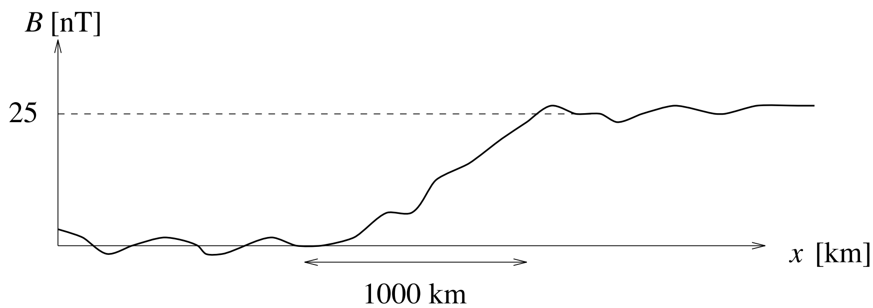

Figure 3:

Part B

5. There are suggestions that NASA should restart the lunar program and go back to the Moon. They plan is to use the Space Launch System (SLS) which is a successor of the Saturn V rocket. SLS is a three stage rocket whose Δv of stage 1 is 4.5 m/s, and Δv of stage 2 is 3.5 km/s. For stage 3: the mass without fuel is 4,000 kg, the fuel mass is 30,000 kg, and the exhaust velocity is 4 km/s. Will they be able to overcome the escape velocity of Earth if they are to carry 80,000 kg of payload? The mass of Earth is $5.98 \cdot 10^{24}$ kg, its mean radius is 6371.2 km, and the gravitational constant is $6.67 \cdot 10^{-11}$ $\mathrm{Nm^2/kg^2}$. (4 p)

6. Consider an auroral electron (energy in the 10 keV range) with pitch angle α at some point on a field line in the auroral zone. Energy and magnetic moment due to gyro motion (the first adiabatic invariant) are both conserved.

   (a) Show that the particle is moving on the surface of a magnetic flux tube (which means that you shall show that the total magnetic flux inside the particle gyroorbit is constant). (2 p)

   (b) Derive a condition on α and the magnetic field strengths (locally and down in the atmosphere) that must be satisfied for the particle to reach the atmosphere before it is mirrored. (2 p)

7. When passing the subsolar point, which is the point where the magnetopause is intersected by the Sun-Earth line, a spacecraft records a magnetic field as in Figure 3.

   (a) Estimate the current density ($\mathrm{A/m^2}$) in the magnetopause layer (2 p).

   (b) Estimate the solar wind number density ($\mathrm{m^{-3}}$), if the solar wind speed was 200 km/s. (1 p)

   (c) Estimate the stand-off distance, i.e. the distance of the magnetopause from the center of the Earth. (2 p)

   (d) Estimate the total solar wind dynamic pressure on the Earth's magnetosphere. (1 p)

   One may assume that the geomagnetic field is described by a dipole field all the way out to the magnetopause, that the interplanetary field as well as thermal pressure may be neglected, and that all solar wind ions are protons.

8. A satellite orbits the Earth in a circular eastward equatorial orbit 1 $\mathrm{R_E}$ above the ground.

   (a) What do we call the plasma region through which the spacecraft is travelling? What velocity (relative to an inertial frame anchored in the Earth) do you expect the plasma to have here? Give a numerical value, with direction. (2 p)

   (b) If the electric field in the rest frame of the plasma is zero, what is the electric field (magnitude and direction) observed by an instrument on the satellite? The geomagnetic field can be approximated with a dipole field with axis along Earth's rotation axis and strength 30 μT on the ground at the equator. (4 p)

Lycka till!

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Galilean transformations:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{B}' = \mathbf{B}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2} m v_\perp^2 / B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu \nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v}_\mathbf{F} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_\perp / v_\parallel$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0}\frac{e^{-r/\lambda_\mathrm{D}}}{r}$$

Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\epsilon_0 K T}{n e^2}}$$

Plasma frequency:

$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{n e^2}{\epsilon_0 m_\mathrm{e}}}$$

Rocket thrust:

$$T = v_\mathrm{e}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_\mathrm{sp} = \frac{\int T\,\mathrm{d}t}{m_\mathrm{fuel} g} = v_\mathrm{e}/g$$

The rocket equation:

$$\Delta v = -g t_\mathrm{burn} + v_\mathrm{e} \ln\left(1 + \frac{m_\mathrm{fuel}}{m_\mathrm{payload+structure}}\right)$$

Emitted thermal radiation power:

$$P_\mathrm{e} = \varepsilon\sigma A_\mathrm{e} T^4$$

Absorbed solar radiation power:

$$P_\mathrm{a} = \alpha A_\mathrm{a} I_\mathrm{rad}$$

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