# Tentamen för kursen Rymdfysik (1FA255)
# 2013-10-23

Uppsala universitet
Institutionen för fysik och astronomi
Avdelningen för astronomi och rymdfysik
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Answers should be provided in Swedish or English.

Time: 8:00 - 13:00

Allowed tools: Mathematics Handbook (or equivalent), Physics Handbook, enclosed tables and formula sheets, calculator. A bilingual dictionary, for example English-Swedish or English-German, may also be used.

The exam has two parts:

- **Part A** must be satisfactorily solved in order to pass the course. However, if you have passed some of the examlets 1-4, you do not have to solve the corresponding problem in Part A. Which of the examlets you have passed was stated in an e-mail to you in October 22: it is your responsibility to correctly remember this. This part is only graded by pass/fail.
- **Part B** must be solved if you wish to get a higher grade than pass (3 in the Swedish 3-4-5 system, E in ECTS). Grades will depend on the number of points you score on this part.

## Part A

1. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

   i. Debye shielding means that the charge of an electron only propagates along the magnetic field.
   ii. As seen in an inertial frame, the solar wind flows radially into the sun.
   iii. If the magnetic field is frozen into the plasma, two plasma elements which at one time are on the same magnetic field lines will always be so.
   iv. If the magnetic field varies on time scales shorter than the gyrofrequency, it is not frozen into the plasma.
   v. If the magnetic field varies on spatial scales shorter than the gyroradius, it is not frozen into the plasma.

   (b) Figure 1 shows 95 minutes of magnetic field measurements from the ACE spacecraft in the solar wind close to the Earth but well outside the bow shock. Discuss the magnetic field observation in relation to the Parker spiral, and estimate the electric field (magntitude and direction) an electric field instrument on ACE would likely have measured (if it existed – there is no electric field instrument on ACE).

2. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

   i. The plasmasphere is the region in the geomagnetic tail where the cross-tail current flows.
   ii. The Van Allen radiation belts contain high energy particles magnetically trapped around the Earth.

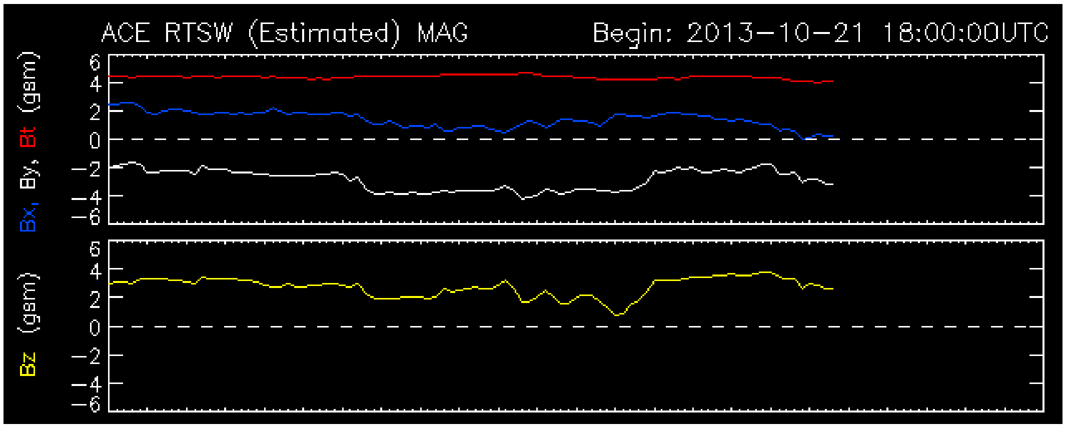


Figure 1: ACE magnetic field data. The xyz components are given in the GSE system. Bt is the magnitude of the magnetic field. The plot covers 2 hours, with data available for about 95 minutes.

iii. The mirror point of a charged particle in a dipole field is the point where the gyroradius is equal to the distance to the centre of the planet.

iv. The magnetic moment of a charged particle due to its gyromotion is an adiabatic invariant.

v. The pitch angle $\alpha$ of a charged particle moving in a dipole field is related to the magnetic latitude $\lambda$ by $\alpha + \lambda = 90°$.

(b) Calculate the gradient drift speed of an electron with kinetic energy 1 keV and pitch angle 90° at geocentric distance 2 $R_{\mathrm{E}}$ in the Earth's magnetosphere.

3. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

i. From the equator, you can only launch directly into orbits with zero inclination.

ii. The geostationary orbit is at about 36,000 km altitude.

iii. For a Kepler orbit, the closest point to the central mass (planet or star or whatever) is called the apoapsis.

iv. To raise the perigee of a satellite orbit in an economical way, one should fire a thruster (rocket) on a satellite when it is at the perigee, in a direction such that the resulting force on the satellite is along its direction of motion.

v. A spacecraft with $\alpha/\epsilon = 1$ at 1 AU will be hotter than an otherwise similar spacecraft with $\alpha/\epsilon = 3$ at 2 AU.

(b) Calculate the altitude of a satellite in circular orbit around the Earth if the period is 2 hours.

4. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

i. An magnetosphere will form around any planet with an atmosphere and some source of ionizing radiation, for example UV radiation from the star it is orbiting.

ii. Around the Earth, currents flow more easily perpendicular to the magnetic field in the ionosphere than in most parts of the magnetosphere.

iii. The aurora is mainly created by electrons accelerated in regions where Birkeland currents flow.

iv. Geomagnetic storms are triggered by solar wind disturbances, often due to coronal mass ejections or solar flares.

v. Dissociative recombination is an important ionization source in the Earth's ionosphere.

(b) Figure 4b shows 24 hours of data from the ACE spacecraft in the solar wind close to Earth (but well outside the bow shock). In which part of this interval do you think magnetospheric substorms were most likely to occur?

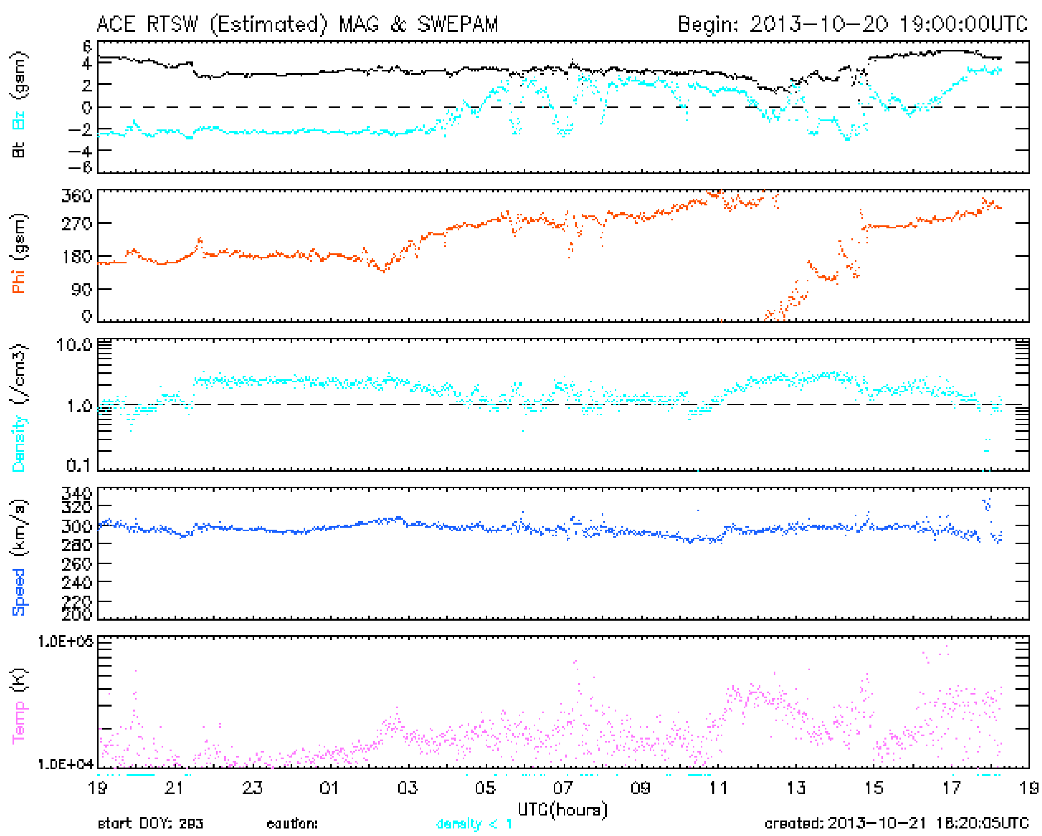
ACE RTSW (Estimated) MAG & SWEPAM
Begin: 2013-10-20 19:00:00UTC
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Figure 2:

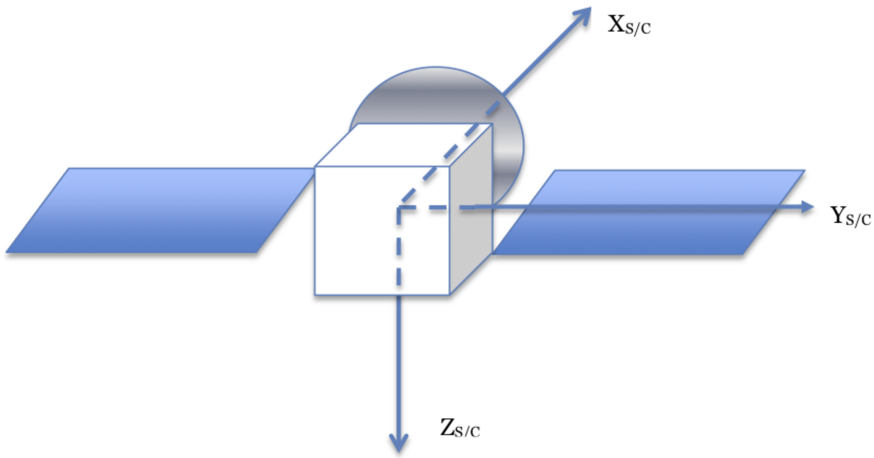


Figure 3: Possible JUICE spacecraft configuration. The blue planes are the solar panels, the grey disk the high gain antenna for communication with the Earth. Image credit: ESA.

## Part B

5. ESA is preparing the Jupiter Icy Moons Explorer, JUICE, for launch in 2022 and arrival at Jupiter in 2030. Assume the spacecraft body to be a cube, 2 m on each side (Figure 3), with perfect internal heat conductivity and covered with a material with absorption coefficient 0.3. Assume there is no heat exchange between the spacecraft body, the solar panels and the high gain antenna. If the spacecraft body temperature is 30°C at Earth orbit when the $Z_{s/c}$ is directed to the Sun and there is no internal heating (all electrical systems turned off), how much internal heating (power in watts) by onboard systems would be needed to keep it at the same temperature when at Jupiter (for the same pointing of the s/c axes)? (3 p)

6. Consider a 1 keV electron with pitch angle 30° in the equatorial plane at 2 AU geocentric distance.
   (a) What is its gyroradius at the mirror point? (2 p)
   (b) At what altitude is its mirror point? Give the answer in kilometers, with two significant digits (like $8.4 \cdot 10^3$ km). (3 p)

7. Consider the following model of the magnetic field in the central part of the geomagnetic tail:

$$\mathbf{B}(\mathbf{r}) = \begin{cases} -B_0\hat{\mathbf{x}} & , \quad z < -a \\ B_0\,\hat{\mathbf{x}}\,\frac{3\,a^2\,z - z^3}{2\,a^3} & , \quad -a \le z \le a \\ B_0\hat{\mathbf{x}} & , \quad z > a \end{cases}$$

   where $B_0 = 30$ nT, $a = 3000$ km and the coordinates are defined as in Figure 4.
   (a) Calculate the current density $\mathbf{j}(\mathbf{r})$ and the magnetic force density $\mathbf{j}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})$ (magnitudes and directions as functions of position). Also calculate their numerical values at $z = 0$. (3 p)
   (b) How can such a current be sustained? Sketch possible orbits of ions and electrons in the $y$-$z$-plane – that is, not the plane of Figure 4, but a plane cutting it at right angles at $x = 0$, and tell why the trajectories look as you have drawn them, with reference to some relevant equation. (2 p)
   (c) Now consider what happens if an instability appears in the region $-a < x < a$, $-10\,a < y < 10\,a$, $-a < z < a$ so that the resistivity in this region includes drastically. When the currents cannot flow through this region as before, where will they close instead? Is this example relevant for any phenomenon in Earth's magnetosphere? (2 p)

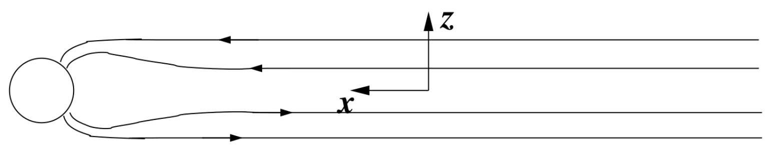


Figure 4: Idealized geometry of the relevant part of the geomagnetic tail (around the origin).

8. Consider the solar wind around 23:00 in Figure 4b.

   (a) When do you expect the plasma observed here by ACE to reach Jupiter orbit (5.2 AU heliocentric distance)? (1 p)

   (b) Assuming the sizes of Jupiter's and Earth's magnetospheres are determined by similar pressure balances between inside magnetic pressure from a roughly dipolar magnetic field and the dynamical pressure in the solar wind, estimate the stand-off distances (the distance from the centre of the planet to the point on the magnetopause on the sun-planet line) for the solar wind ACE saw at 23:00 – that is, use the values you see in the plot for the standoff distance for the Earth, and for Jupiter what you calculate with some reasonable assumption that the values will be when this particular volume of solar wind reaches the giant planet. The radii of Earth and Jupiter are 6,371 km and about 70,000 km, respectively, and the surface magnetic field strengths at the equators about 30 $\mu$T and 0.4 mT. (4 p)

*Lycka till!*

# Space Physics Formulas:
# Complement to Physics Handbook

Charge density in plasma with charge particle species $s$:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Galilean transformations:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{B}' = \mathbf{B}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_{\mathrm{c}} = \omega_{\mathrm{c}}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_{\perp}^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu\nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v}_{\mathbf{F}} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_{\perp}/v_{\parallel}$$

Electrostatic potential from charge $Q$ in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0}\frac{e^{-r/\lambda_{\mathrm{D}}}}{r}$$

Debye length:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_{\mathrm{p}} = \omega_{\mathrm{p}}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_{\mathrm{e}}}}$$

Rocket thrust:

$$T = v_{\mathrm{e}}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_{\mathrm{sp}} = \frac{\int T\,\mathrm{d}t}{m_{\mathrm{fuel}}g} = v_{\mathrm{e}}/g$$

The rocket equation:

$$\Delta v = -gt_{\mathrm{burn}} + v_{\mathrm{e}}\ln\left(1 + \frac{m_{\mathrm{fuel}}}{m_{\mathrm{payload+structure}}}\right)$$

Emitted thermal radiation power:

$$P_{\mathrm{e}} = \varepsilon\sigma A_{\mathrm{e}}T^4$$

Absorbed solar radiation power:

$$P_{\mathrm{a}} = \alpha A_{\mathrm{a}}I_{\mathrm{rad}}$$

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