Tentamen för Rymdfysik I
2007-06-11

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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well. Answers may of course be given in Swedish or English, according to your own preference.

Time: 13:00 - 18:00

Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet, calculator. A bilingual dictionary, for example English-Swedish or English-German, may also be used. German equivalents of Mathematics and Physics Handbook are accepted.

1. Here follows a set of multiple choice questions, where you must find out which statements are correct. For each question (1-1, 1-2 etc), there is only one correct combination of answers, say ”A and B” or ”none”. To score on a question, you need to have exactly the right combination. You are welcome to add comments to your answers. Any number of alternatives can be correct (0 – 3). (1 p/question, 10 p in total)

1:1. Satellite orbits:

A. To raise the perigee of a satellite orbit, one can fire a thruster (rocket) on a satellite when it is at the perigee, in a direction such that the resulting force on the satellite is along the direction of motion.

B. It is possible to change the energy of an interplanetary spacecraft by letting it pass close to a planet.

C. Any geostationary orbit must be equatorial.

1:2. Rockets and propulsion:

A. For rocket launches from Earth, the only important parameter is the total impulse of the rocket, $\int F, \mathrm{d}t$, not the force F itself.

B. Rockets are often launched westwards to take advantage of the rotation of the Earth.

C. Launch sites at high latitudes cannot easily launch satellites into equatorial orbits (inclination $\sim 0°$).

1:3. Spacecraft systems:

A. Satellites around the Earth communicates with ground by means of radio transmitters and receivers. Because of the large distances involved, radio waves cannot be used for communication with interplanetary spacecraft.

B. The most commonly used means of generating electricity in space is by use of small thermonuclear reactors.

C. To achieve a stable attitude (orientation) in space, many spacecraft use two or more spinning wheels on orthogonal axes. Turning the spacecraft around some axis, for example for re-directing a rigid antenna, can then be done by simply slowing down or spinning up the wheel spinning around that axis for a short while and then return it to the original state of rotation.

1:4. Magnetospheric dynamics:

A. Figure 4 shows 24 hours of solar wind data gathered by the ACE spacecraft, situated at the L1 Lagrange point about 1,500,000 km from the Earth in the direction of the sun. Negative B_z in the solar wind implies that we can get reconnection on the dayside magnetopause, and thus large energy input to the magnetosphere. Hence, the period after 13:00 UTC should be more active than the rest of these 24 hours.

B. Reconnection on the dayside leads to transport of magnetic flux to the geomagnetic tail. After a period of storing magnetic energy in the tail in this way, large amounts of energy can be rapidly released in geomagnetic substorms.

C. Geomagnetic storms are caused directly by solar wind disturbances. In contrast, geomagnetic substorms are not necessarily triggered by direct solar wind disturbances, but are triggered by instabilities in the magnetosphere itself when a large amount of magnetic energy is stored in the tail.

1:5. Motion of charged particles:

A. The gyromotion of electrons and positive ions have the same direction (negative sense of rotation around $\mathbf{B}$) – otherwise, there would flow enormous currents in the plasma.

B. The orbital magnetic moment of an electron moving in a magnetic field is conserved if the field varies only a little during an electron gyroperiod or inside an electron gyroradius.

C. If an electric field is applied perpendicular to the magnetic field in a plasma, all plasma particles drift in the same direction, irrespective of charge.

1:6. Plasmas:

A. A plasma is a gas of charged particles.

B. The magnetic pressure is $B^2/(2\mu_0)$, without any reference to any plasma parameter: hence, magnetic pressure gradients are equally important for a neutral gas and a plasma.

C. The electric field in the frame of reference of the plasma is close to zero (for processes on sufficiently large scales in time and space).

1:7. Ionosphere:

A. The electron density in the dayside ionosphere is determined by the solar UV intensity and the Earth's atmospheric density.

B. Due to collissions between particles, the conductivity in the direction parallel to the magnetic field is lower in the ionosphere than in the magnetosphere.

C. Field-aligned currents can flow along the magnetic field from the magnetic tail into the ionosphere, flow perpendicular to $\mathbf{B}$ in the ionosphere, and then flow up along the magnetic field lines back into another part of the magnetosphere.

1:8. Aurora:

A. The aurora is mainly caused by electrons from the sun, hitting the Earth's atmosphere at the poles.

B. The aurora is mainly caused by electrons accelerated inside the Earth's magnetosphere in regions where field-aligned currents flow.

C. Auroral arcs are ususally elongated in the north-south direction.

1:9. Magnetosphere. The letters below refer to labels in Figure 1.

A. The solar wind (a) is deflected by the bow shock (b) and cannot enter the magnetosheath (c).

B. The dayside (d) and nightside (e) radiation belts that look like separate banana-shaped features in this plot are really the same region, which in reality is toroidal (doughnut shaped), extending all around the Earth around the equator.

C. The magnetopause (f) separates the plasmasphere (c) from the ionosphere.

1:10. Sunspots and solar activity. Figure 2 shows a summary of the sunspot statistics for the last 15 years.

A. Sunspots look dark because they are cooler than the surrounding material.

B. The sunspot number is a proxy for the general solar activity: the less sunspots there are in a given month, the more active is the sun.

C. The next solar maximum can be expected to appear around 2008.

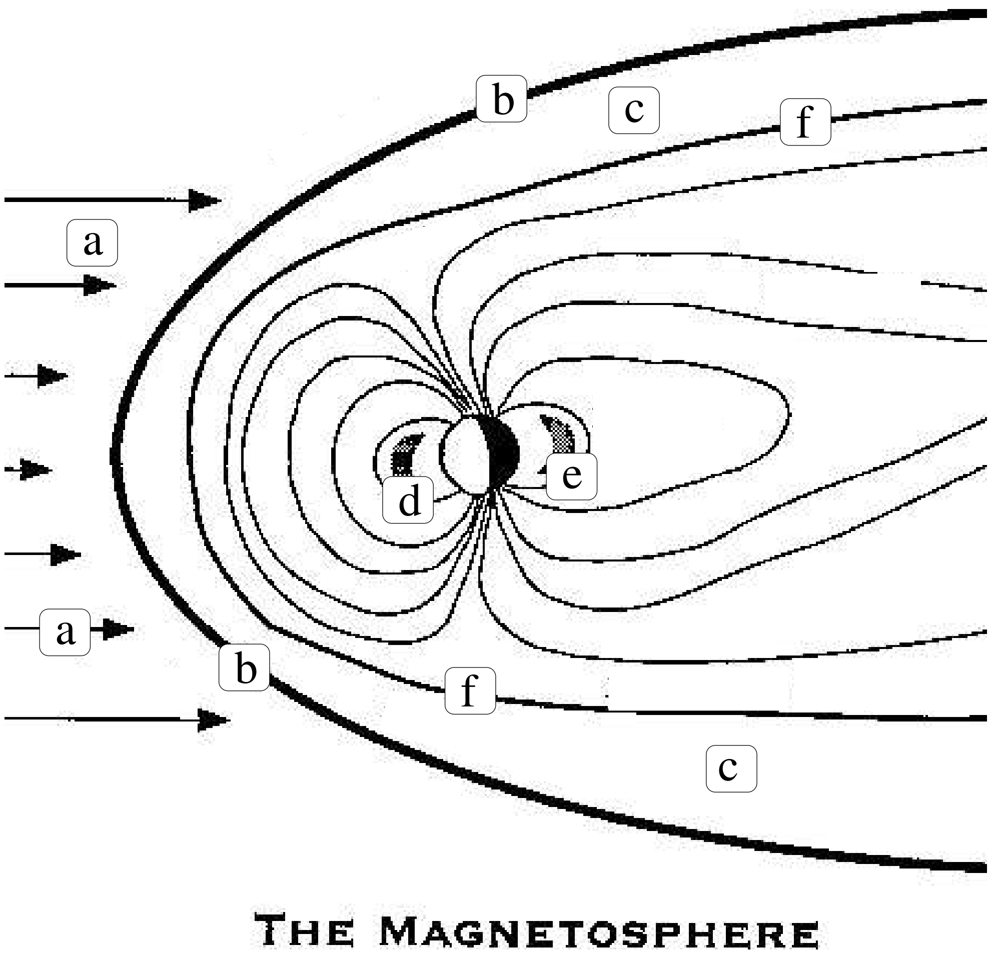

Figure 1:

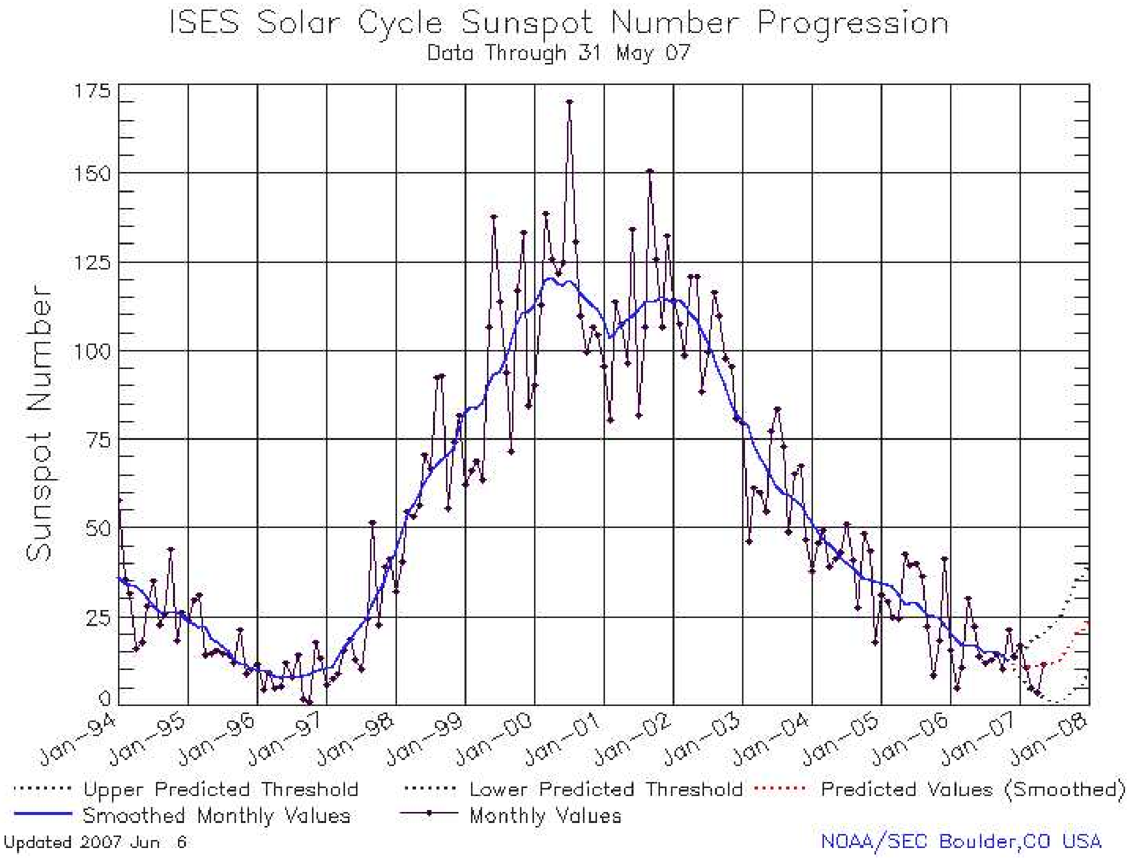

Figure 2: Recent sunspot statistics.

2. At a position where the magnetic field is 2 % as strong as it is in the ionosphere on the same field line, an electron initially having 20 eV kinetic energy and a pitch angle of 45° is accelerated downward by an electric field directed exactly along the magnetic field, so that only the parallel part of the kinetic energy is affected. The total potential increase seen by the electron is 1 kV, and the electric field is concentrated to a very small region of space.

   Calculate the electron kinetic energies due to its motion parallel and perpendicular to $\mathbf{B}$, $mv_{\|}^2/2$ and $mv_{\perp}^2/2$, before and after the acceleration by the potential drop. Can the electron reach the ionosphere and contribute to the aurora? (5 p)

3. The sensor of our instrument (a kind of space weather station known as a Langmuir probe) on the Cassini spacecraft, now orbiting Saturn, is a sphere ($r = 25$ mm) covered with titanium nitride, with absorption coeffiecient 0.47 and emission coefficient 0.10. Since its launch in 1997, Cassini has been as close to the sun as Venus (0.72 AU) and as far out as Saturn (9.54 AU). What is the temperature range we had to design the sensor for, i.e. what is the lowest and highest temperatures we expect the probe should reach? (3 p)

4. Consider the following model of the magnetic field in the central part of the geomagnetic tail:

$$\mathbf{B}(\mathbf{r}) = \begin{cases} -B_0\hat{\mathbf{x}} & , \quad z < -a \\ B_0\,\hat{\mathbf{x}}\,\frac{3\,a^2\,z - z^3}{2\,a^3} & , \quad -a \le z \le a \\ B_0\hat{\mathbf{x}} & , \quad z > a \end{cases}$$

   where $B_0 = 10$ nT, $a = 2000$ km and the coordinates are defined as in Figure 3.

   (a) Calculate the current density $\mathbf{j}(\mathbf{r})$ and the magnetic force density $\mathbf{j}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})$ (magnitudes and directions as functions of position). Also calculate their numerical values at $z = 0$. (3 p)

   (b) How can such a current be sustained? Sketch possible orbits of ions and electrons in the y-z-plane – that is, not the plane of Figure 3, but a plane cutting it at right angles at $x = 0$, and tell why the trajectories look as you have drawn them, with reference to some relevant equation. (2 p)

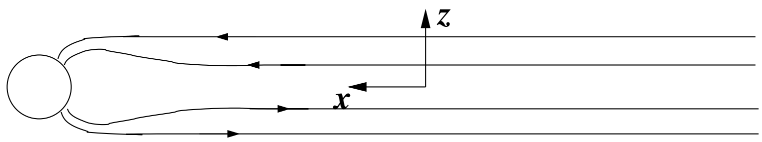

Figure 3: Idealized geometry of the relevant part of the geomagnetic tail (around the origin).

5. Figure 4 shows 24 hours of solar wind data gathered by the ACE spacecraft, situated at the L1 Lagrange point about 1,500,000 km from the Earth in the direction of the Sun. Vectors are given in GSM coordinates, which for the purposes of this problem can be seen as approximately the same as GSE coordinates: x thus points to the Sun, and z along the Earth's angular momentum because of its motion around the Sun. UTC is coordinated universal time.

   (a) How does the direction of the magnetic field at 22:00 UTC compare to what you would typically expect for the interplanetary magnetic field? Explain your answer. (2 p)

   (b) If ACE had an electric field instrument (it has not), what electric field would have been measured at 22:00 UTC? Calculate all three components of the electric field vector, assuming that the solar wind flows in the $-x$ direction. What else did you have to assume? (3 p)

   (c) Estimate the distance from the Earth's centre to the subsolar point on the magnetopause at 22:00 UTC. The geomagnetic field may be assumed to be a dipole field of strength 30 μT at the equator, and you may neglect the solar wind travel time from ACE to Earth. (2 p)

Lycka till!

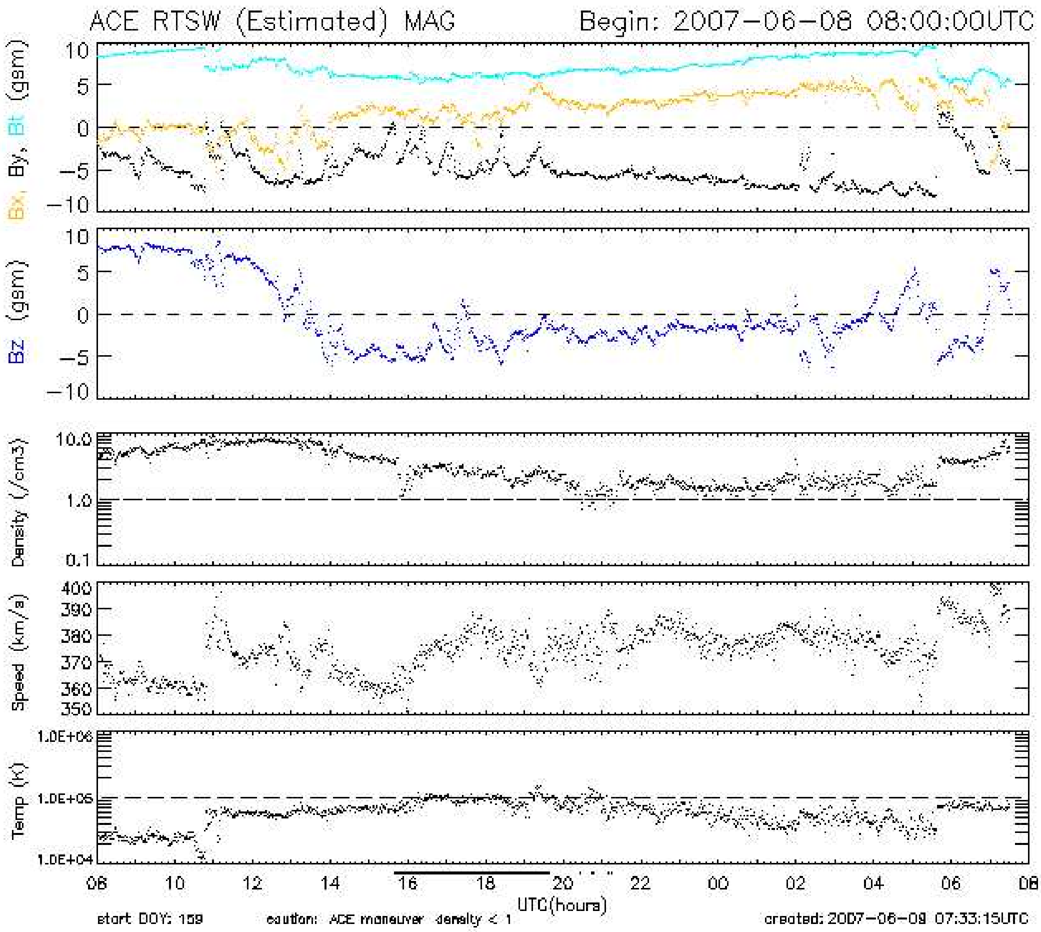

Figure 4: ACE solar wind data. In the top plot, the B_x component (red) is the centre curve around 22:00.

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \tfrac{ne}{B}\left(\tfrac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \tfrac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \tfrac{ne}{B}\left(\tfrac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \tfrac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\tfrac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \tfrac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2} m v_\perp^2 / B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu \nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v_F} = \frac{\mathbf{F} \times \mathbf{B}}{q B^2}$$

Pitch angle:

$$\tan \alpha = v_\perp / v_\parallel$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_\mathrm{D}}}{r}$$

Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\epsilon_0 K T}{n e^2}}$$

Plasma frequency:

$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = \frac{1}{2\pi} \sqrt{\frac{n e^2}{\epsilon_0 m_\mathrm{e}}}$$

Rocket thrust:

$$T = v_\mathrm{e} \frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_\mathrm{sp} = \frac{\int T \, \mathrm{d}t}{m_\mathrm{fuel} g} = v_\mathrm{e}/g$$

The rocket equation:

$$\Delta v = -g t_\mathrm{burn} + v_\mathrm{e} \ln \left(1 + \frac{m_\mathrm{fuel}}{m_\mathrm{payload+structure}} \right)$$

Emitted thermal radiation power:

$$P_\mathrm{e} = \varepsilon \sigma A_\mathrm{e} T^4$$

Absorbed solar radiation power:

$$P_\mathrm{a} = \alpha A_\mathrm{a} I_\mathrm{rad}$$