Tentamen för Rymdfysik I
2007-03-09

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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well. Answers may of course be given in Swedish or English, according to your own preference.

Time: 15:00 - 20:00

Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet, calculator. A bilingual dictionary, for example English-Swedish or English-German, may also be used. German equivalents of Mathematics and Physics Handbook are accepted.

1. Here follows a set of multiple choice questions, where you must find out which statements are correct. For each question (1-1, 1-2 etc), there is only one correct combination of answers, say ”A and B” or ”none”. To score on a question, you need to have exactly the right combination. You are welcome to add comments to your answers. Any number of alternatives can be correct (0 – 3). (1 p/question, 10 p in total)

   1:1. Satellite orbits:

      A. To raise the perigee of a satellite orbit, one can fire a thruster (rocket) on a satellite when it is at apogee, in a direction such that the resulting force on the satellite is along the direction of motion.

      B. The speed of satellite in circular orbit decreases with increasing radius of the orbit. Hence, the total energy also decreases with increasing radius, so to get a larger orbit, one needs to brake by firing a such that the force is opposite to the direction of motion.

      C. The geostationary orbit is possible thanks to inhomogeneities in the Earths gravitational field.

   1:2. Solar activity:

      A. The activity of the sun varies in an 11 -year cycle. If taking into account the magnetic polarity of the sun, the period is 22 years.

      B. The intensity of visible light changes very little during a solar cycle.

      C. The sunspot number correlates with other forms of solar activity: more sunspots means the sun is more active also in other ways.

   1:3. Rockets and propulsion:

      A. For rocket launches from Earth, the only important parameter is the total impulse of the rocket, $\int F, \mathrm{d}t$, not the force F itself.

      B. Rockets are often launched eastwards to take advantage of the rotation of the Earth.

      C. Launch sites at low latitudes cannot easily launch satellites into polar orbits (inclination $\sim 90°$).

   1:4. Space weather:

      A. Geomagnetic storms are usually worse than substorms in terms of their impact on human systems (satellites, power grids, ...).

      B. Geomagnetic storms are driven by perturbations in the solar wind, ultimately originating in the Sun.

      C. The radiation belts almost disappear during geomagnetic storms.

1:5. Spaceflight:

A. The main reason to use several rocket stages is that it is difficult to construct sufficiently powerful rockets to concentrate everything into a single stage.

B. The only use of planetary swing-by maneouveres is to change the course of your spacecraft without having to use any fuel. As the gravitational field is conservative, there can be no gain of speed during a planetary swing-by.

C. Spacecraft must be designed to withstand heavy vibrations in order to survive passages through the Earth's bow shock.

1:6. Motion of charged particles:

A. The gyromotion of electrons and positive ions have the same direction (negative sense of rotation around $\mathbf{B}$) – otherwise, there would flow enormous currents in the plasma.

B. The orbital magnetic moment of an electron moving in a magnetic field is conserved if the field varies only a little during an electron gyroperiod or inside an electron gyroradius.

C. The ∇B drift causes electrons to flow eastward and positive ions westward around the Earth.

1:7. Plasmas:

A. A plasma is a gas of charged particles.

B. The magnetic pressure is $B^2/(2\mu_0)$, without any reference to any plasma parameter: hence, magnetic pressure gradients are equally important for a neutral gas and a plasma.

C. The magnetic field in the frame of reference of the plasma is close to zero (for processes on sufficiently large scales in time and space).

1:8. Ionosphere:

A. The electron density in the dayside ionosphere is determined by the solar UV intensity and the Earth's atmospheric density.

B. Due to collissions between particles, the conductivity in the direction parallel to the magnetic field is lower in the ionosphere than in the magnetosphere.

C. Field-aligned currents can flow along the magnetic field from the magnetic tail into the ionosphere, flow perpendicular to $\mathbf{B}$ in the ionosphere, and then flow up along the magnetic field lines back into another part of the magnetosphere.

1:9. Aurora:

A. The aurora is mainly caused by electrons from the sun, hitting the Earth's atmosphere at the poles.

B. The aurora is mainly caused by electrons accelerated inside the Earth's magnetosphere in regions where field-aligned currents flow.

C. Auroral arcs are ususally elongated in the north-south direction.

1:10. Magnetosphere. The letters below refer to labels in Figure 1.

A. The solar wind (a) abruptly slows down at the bow shock (b) to become a more turbulent flow in the magnetosheath (c).

B. The dayside (d) and nightside (e) radiation belts are two distinct banana-shaped regions filled mainly with electrons and ions, respectively, and separated by empty regions around the dawn and dusk directions.

C. The magnetopause (f) separates the shocked solar wind plasma in the magnetosheath from the more tenous plasma in the magnetosphere.

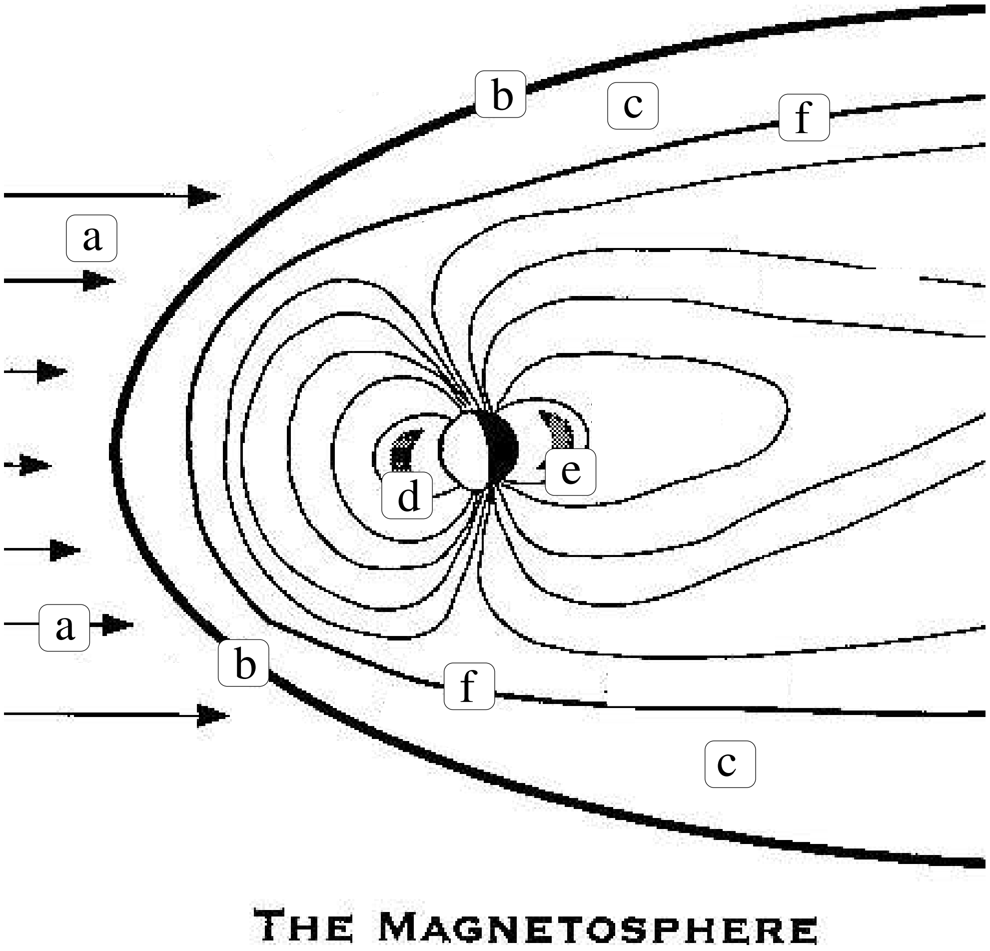

Figure 1:

2. At a position where the magnetic field is 2 % as strong as it is in the ionosphere on the same field line, an electron initially having 20 eV kinetic energy and a pitch angle of 45° is accelerated downward by an electric field directed exactly along the magnetic field, so that only the parallel part of the kinetic energy is affected. The total potential increase seen by the electron is 1 kV, and the electric field is concentrated to a very small region of space.

   (a) What is the electron kinetic energies due to its motion parallel and perpendicular to $\mathbf{B}$, $mv_{\|}^2/2$ and $mv_{\perp}^2/2$, before and after the acceleration by the potential drop? (2 p)

   (b) Can the electron reach the ionosphere and contribute to the aurora? (3 p)

3. Figure 2 shows recent data from the ACE spacecraft, situated between the Earth and the Sun about 1.5 million km from the Earth. The four plots in the figure are explained in the figure text.

   (a) From (some of) these data, make a quantitative argument for who is in main control of the dynamics of the solar wind: is it the magnetic field which decides the flow, or is it the flow that controls the magnetic field? (3 p)

   (b) Can you locate any time interval when you think it is more likely that the Earth's magnetosphere interacted strongly with the solar wind by reconnection on the dayside magnetopause, ultimately leading to geomagnetic substorms? Describe your argument, possibly with a figure. Do you need to take the solar wind propagation time from ACE to Earth into account in your answer? (2 p)

   (c) At 2007-03-03 00:00 (i.e. at the label '03' in the figure), the complete $\mathbf{B}$ vector (not shown) was $(-3, 2, 2)$ nT. Calculate the electric field $\mathbf{E}$ (in mV/m, all three components) that ACE would have measured if it had had an electric field instrument onboard. (2 p).

4. You are given the task to design a cheap spacecraft to Mars. The spacecraft is a cylinder of radius 1 m and height 1 m. It will be spinning around its axis, and thus have solar panels mounted all around its mantle surface. For reasons of power efficiency, the axis of the cylinder must be kept orthogonal to the line between the spacecraft and the sun. To keep operational costs and power consumption down, the spacecraft is to be turned off during all the passage from Earth to Mars, without even any heaters running, but must still have its axis perpendicular to the line to sun so as not to cause any expensive maneouvres. Your only way to control the temperature is to paint the surfaces (the solar panels on the mantle areas as well as the circular ends) with a transparent paint, which for economical reasons has to be the same on all surfaces. The spacecraft carries some intricate laboratory equipment for analyzing the Martian atmosphere, and this equipment, and hence all the spacecraft, must be kept at temperatures between +7°C and +87°C. If α and ϵ are the absorbtion and emission coefficients of your transparent paint, what range of values of α/ϵ is allowed? Is there such a range at all, or is it impossible to construct the spacecraft according to this specification? Mars is 1.52 times further from the sun than the Earth is. (4 p)

5. The diagram in Figure 3 shows the altitude distribution of some neutral atmospheric constituents, and the day and night profiles of the electron density in the Earth's ionosphere.

   (a) Derive (from the equation of motion of a neutral gas and an assumption of constant gravitational field) an expression showing why the concentrations of neutral molecules decrease approximately exponentially with increasing altitude, and why the concentration of atomic oxygen (O) decreases slower with altitude than the N_2 density, which in turn decreases slower than the concentration of molecular oxygen (O_2). State explicitly all assumptions you make. (2 p)

   (b) Why are the day- and nighttime profiles for the electron density different? (1 p)

   (c) Why is there a maximum in the electron density around 300 km while the neutral density just decreases monotonically with altitude? Why doesn't the ionosphere extend all the way down to ground? (1 p)

Lycka till!

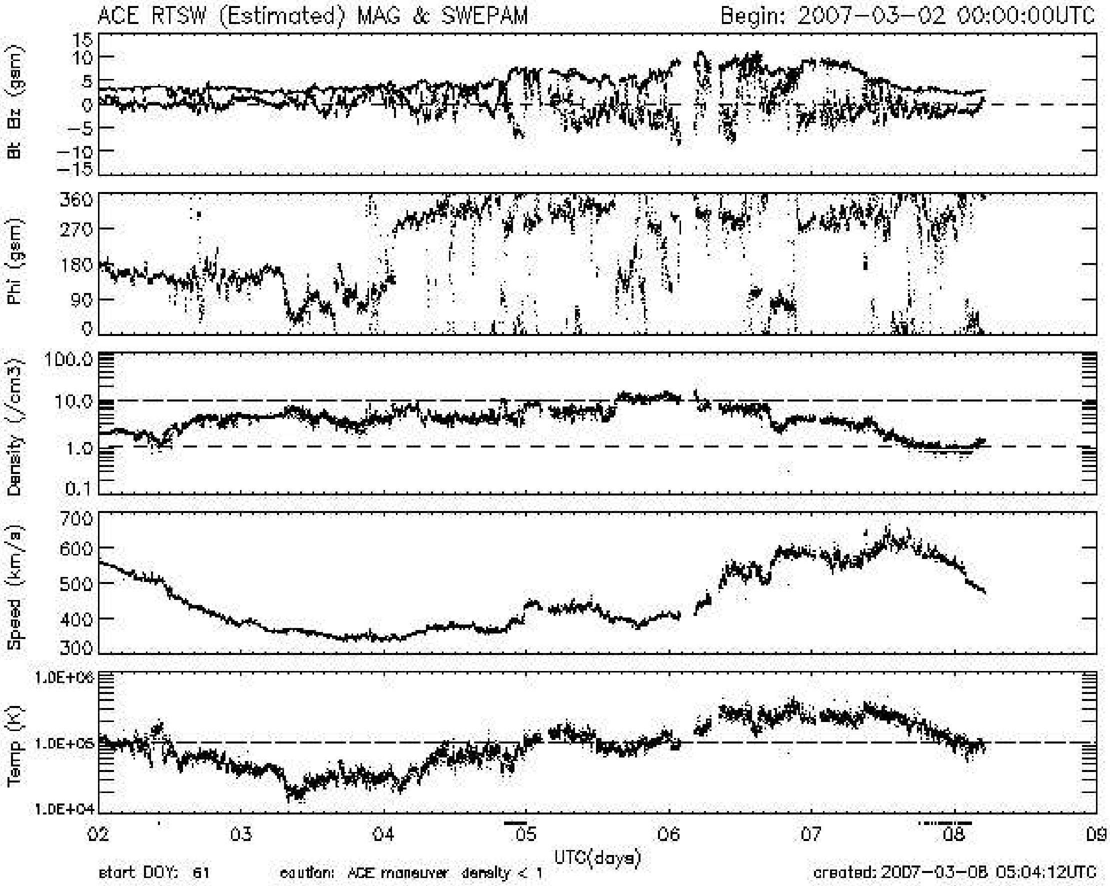

Figure 2: Recent ACE solar wind data. Panels from top to bottom: (1) Interplanetary magnetic field strentgth B_{t} (in nT) and the z-component of the magnetic field B_z (also in nT). Both curves (sets of data points) are black, but you can still realize which is which (how?). The coordinate system used is known as geocentric solar magnetic coordinates (GSM), which for the pursposes of this problem can be taken to be approximately the same as the GSE (geocentric solar ecliptic) coordinates you are used to from the course: thus $\hat{\mathbf{x}}$ points to the sun and $\hat{\mathbf{z}}$ out of the ecliptic toward the ecliptic north pole; $\hat{\mathbf{y}}$ thus is roughly in the direction of the Earth's motion around the Sun. (2) The angle ϕ of the magnetic field in the ecliptic plane, defined so that $\phi = 0$ if $\mathbf{B}$ points along $\hat{\mathbf{x}}$ and $\phi = 90\circ$ if pointing along $\hat{\mathbf{y}}$. (3) Proton number density n (in cm^{-3}). (4) Solar wind flow speed u (in km/s). (5) Proton temperature T (in K).

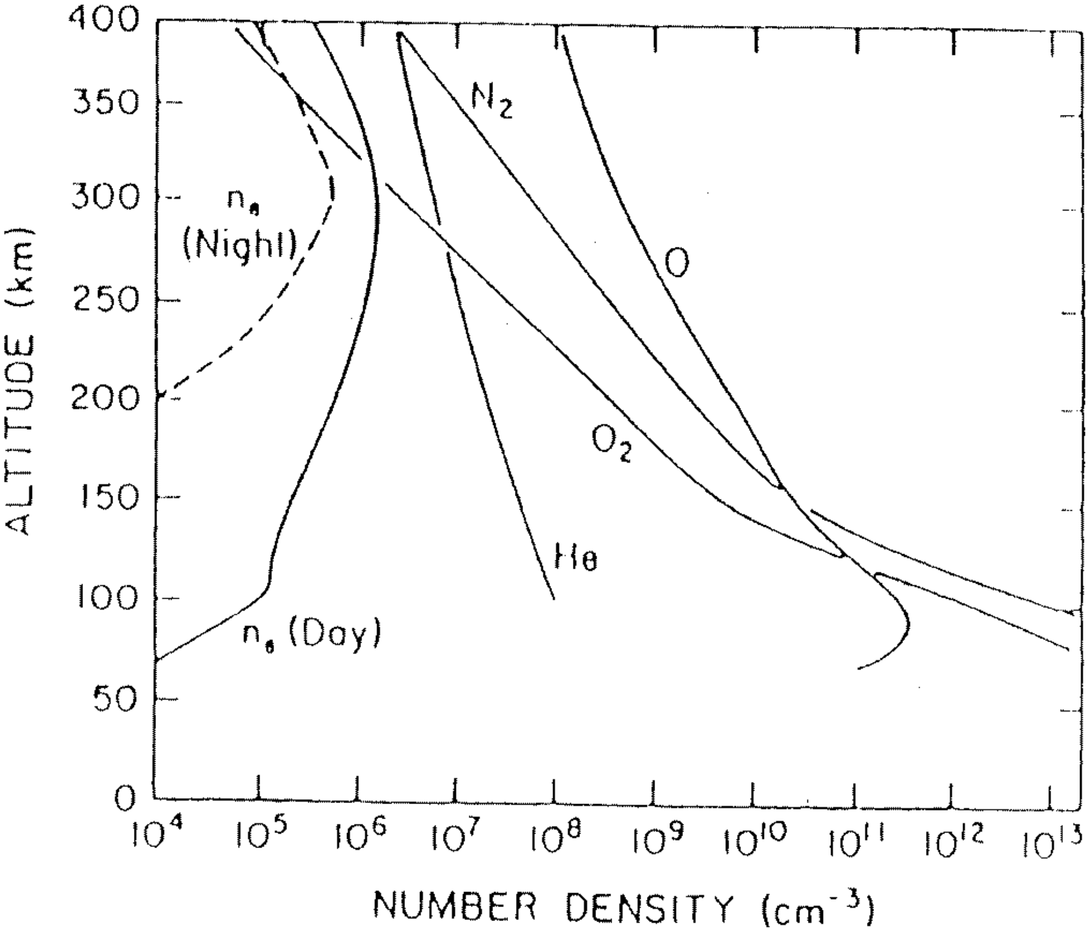

Figure 3: Altitude profiles of some neutral gas components, and of the electron density.

Space Physics Formulas: Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_{\mathrm{m}} \frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_{\mathrm{m}} \frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for ”frozen-in” magnetic field:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Ohm’s law:

$$\mathbf{j} = \begin{pmatrix} \sigma_{\mathrm{P}} & \sigma_{\mathrm{H}} & 0 \\ -\sigma_{\mathrm{H}} & \sigma_{\mathrm{P}} & 0 \\ 0 & 0 & \sigma_{\parallel} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_{\parallel} \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_{\mathrm{P}} &= \tfrac{ne}{B}\left(\tfrac{\omega_{\mathrm{ci}}\nu_{\mathrm{i}}}{\omega_{\mathrm{ci}}^2+\nu_{\mathrm{i}}^2} + \tfrac{\omega_{\mathrm{ce}}\nu_{\mathrm{e}}}{\omega_{\mathrm{ce}}^2+\nu_{\mathrm{e}}^2}\right) \\ \sigma_{\mathrm{H}} &= \tfrac{ne}{B}\left(\tfrac{\omega_{\mathrm{ci}}^2}{\omega_{\mathrm{ci}}^2+\nu_{\mathrm{i}}^2} - \tfrac{\omega_{\mathrm{ce}}^2}{\omega_{\mathrm{ce}}^2+\nu_{\mathrm{e}}^2}\right) \\ \sigma_{\parallel} &= ne^2\left(\tfrac{1}{m_{\mathrm{i}}\nu_{\mathrm{i}}} + \tfrac{1}{m_{\mathrm{e}}\nu_{\mathrm{e}}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_{\mathrm{c}} = \omega_{\mathrm{c}}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2} m v_\perp^2 / B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu \nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v_F} = \frac{\mathbf{F} \times \mathbf{B}}{q B^2}$$

Pitch angle:

$$\tan \alpha = v_\perp / v_\parallel$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_\mathrm{D}}}{r}$$

Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\epsilon_0 K T}{n e^2}}$$

Plasma frequency:

$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = \frac{1}{2\pi} \sqrt{\frac{n e^2}{\epsilon_0 m_\mathrm{e}}}$$

Rocket thrust:

$$T = v_\mathrm{e} \frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_\mathrm{sp} = \frac{\int T \, \mathrm{d}t}{m_\mathrm{fuel} g} = v_\mathrm{e}/g$$

The rocket equation:

$$\Delta v = -g t_\mathrm{burn} + v_\mathrm{e} \ln \left(1 + \frac{m_\mathrm{fuel}}{m_\mathrm{payload+structure}} \right)$$

Emitted thermal radiation power:

$$P_\mathrm{e} = \varepsilon \sigma A_\mathrm{e} T^4$$

Absorbed solar radiation power:

$$P_\mathrm{a} = \alpha A_\mathrm{a} I_\mathrm{rad}$$