Tentamen för Rymdfysik I
2006-08-15

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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well. Answers may of course be given in Swedish or English, according to your own preference.

Time: 09:00 - 14:00

Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet, calculator. A bilingual dictionary, for example English-Swedish or English-French, may also be used.

1. Here follows a set of multiple choice questions, where you must find out which statements are correct. For each question (1-1, 1-2 etc), there is only one correct combination of answers, say ”A and B” or ”none”. To score on a question, you need to have exactly the right combination. You are welcome to add comments to your answers. Any number of alternatives can be correct (0 – 3). (1 p/question, 10 p in total)

   1:1. Reconnection and substorms:

   A. Reconnection is the common mechanism driving explosive magnetic events like solar flares and geomagnetic substorms.
   B. During reconnection, magnetic energy is converted to particle energy in the plasma.
   C. The energy released in a geomagnetic substorm has been stored in the geomagnetic tail.

   1:2. Solar activity:

   A. The activity of the sun varies in a 22-year cycle. If the last solar maximum was around year 2000, the next solar maximum will be around 2022.
   B. The intensity of visible light from the sun drops by around 50% from solar maximum to solar minimum.
   C. The sunspot number anticorrelates with the solar activity: more sunspots means the sun is colder and less active.

   1:3. Rockets and propulsion:

   A. For rocket launches from Earth, the only important parameter is the total impulse of the rocket, $\int F, \mathrm{d}t$, not the force F itself.
   B. Rockets are often launched westwards to take advantage of the rotation of the Earth.
   C. The fraction of mass that is fuel can be as high as 90% for a rocket ready for launch.

   1:4. Magnetosphere:

   A. The magnetosphere disappears at nighttime, because the ionization by solar radiation then stops.
   B. Electrical currents flow on the magnetopause, which is the outer boundary of the magnetosphere.
   C. The radiation belts (van Allen bets) contain trapped energetic protons and electrons.

   1:5. Solar wind:

   A. The solar wind is supersonic (i.e. it blows faster than the sound speed in itself).

B. The solar wind blows out through the solar system in a spiral pattern (as seen in an inertial frame).
C. The typical solar wind speed at Earth orbit is in the range 200 – 600 km/s.

1:6. Motion of charged particles:

A. The orbital magnetic moment of an electron moving in a magnetic field is conserved if the field varies only a little during an electron gyroperiod or inside an electron gyroradius.
B. In a hydrogen plasma in thermodynamic equilibrium, electrons move much faster than protons.
C. An electric field perpendicular to a magnetic field does not give rise to any electric currents in a collissionless plasma.

1:7. Plasmas:

A. A plasma is a gas of charged particles.
B. Plasmas are usually dominated by ions. The electrons are almost always only a small fraction of the total number of particles.
C. The electric field on large scales in space and time is usually related to the flow velocity and the magnetic field by the relation $\mathbf{E} \times \mathbf{v} + \mathbf{B} = 0$.

1:8. Ionosphere:

A. The electron density in the dayside ionosphere is determined by the solar UV intensity and the Earth's atmospheric density.
B. Due to collissions between particles, the conductivity in the direction parallel to the magnetic field is lower in the ionosphere than in the magnetosphere.
C. Field-aligned currents can flow along the magnetic field from the magnetic tail into the ionosphere, flow perpendicular to $\mathbf{B}$ in the ionosphere, and then flow up along the magnetic field lines from a different part of the ionosphere.

1:9. Aurora:

A. The aurora is mainly caused by electrons from the sun, hitting the Earth's atmosphere at the poles.
B. The aurora is mainly caused by electrons accelerated inside the Earth's magnetosphere in regions where field-aligned currents flow.
C. The main reason why auroras are easier to see in Kiruna than in Berlin in winter is that the sky in Kiruna is darker, so that the aurora can easily be detected.

1:10. Which statements about temperatures are correct?

A. The equilibrium temperature of an interplanetary spacecraft, travelling between e.g. Earth and Mars, is determined by the temperature of the solar wind plasma (around 10 eV).
B. The equilibrium temperature of an interplanetary spacecraft, travelling between e.g. Earth and Mars, is determined by the balance between absorbtion of solar radiation and emission of thermal radiation.
C. The equilibrium temperature of an interplanetary spacecraft, travelling between e.g. Earth and Mars, is determined by the cosmic microwave background radiation temperature (around 2.7 K).

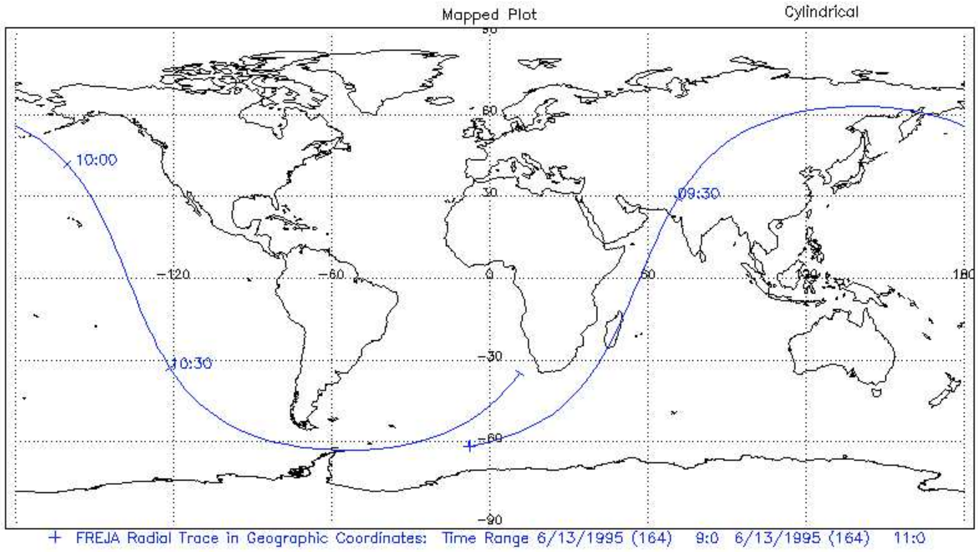

Figure 1: The ground track of Freja between 9:00 and 11:00 UT on June 13, 1995

2. Consider a spacecraft formed as a rectangular box of 2 x 2 x 4 m, with uniform surface properties and good thermal conductivity. At a distance of 1 AU from the sun, the equilibrium temperature of the spacecraft is 30°C when one of the 2 x 2 m surfaces points to the sun and all internal heating sources are turned off. When an electrical equipment producing 15 W was turned on, the temperature rose to 32°C. Calculate the absorbtion and emission coefficients. (3 p)

3. The Swedish-German Freja satellite was launched in October 1992 into a near-circular, eastward orbit around the Earth, with inclination $63°$ and orbital period 1 h 50 min. For the calculations here, we assume the geomagnetic field to be a dipole field of strength 30 μT on the ground at the equator, with dipole axis parallel to the Earth's spin axis.

   (a) The ground track of Freja for two hours is shown in Figure 1. If the orbital period is 1 h 50 min as said above, why isn't the orbit closed in this plot? (1 p)

   (b) Calculate Freja's speed and altitude (height above the Earth), and also give the inclination of the Freja orbit. (2 p)

   (c) What is the strongest magnetic field (in μT) you expect the magnetometer onboard Freja should measure? In addition, mark on the map in Figure 1 where in the orbit this maximal value is seen. (2 p)

   (d) Assume the plasma to be co-rotating with the Earth, and the electric field in the plasma to be zero. What is the value and direction of the electric field measured by the Freja electric field instrument at the point(s) of maximum magnetic field strength? (2 p)

4. Consider an oxygen ion (O^+) with a kinetic energy of 1 MeV and no velocity along the magnetic field, moving in the equatorial plane at a distance of 4 R_{E} from the center of the Earth. Calculate the drift period for this particle, i.e. the time it takes to complete on full orbit around the Earth. The geomagnetic field may be taken to be a dipole field with strength 30 μT on the ground at the equator. (3 p)

5. Consider the following model of the magnetic field in the central part of the geomagnetic tail:

$$\mathbf{B}(\mathbf{r}) = \begin{cases} -B_0\hat{\mathbf{x}} & , \quad z < -a \\ B_0\,\hat{\mathbf{x}}\,\frac{3\,a^2\,z - z^3}{2\,a^3} & , \quad -a \le z \le a \\ B_0\hat{\mathbf{x}} & , \quad z > a \end{cases}$$

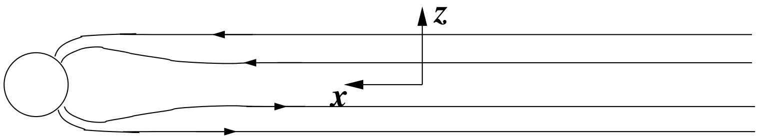

Figure 2: Idealized geometry of the relevant part of the geomagnetic tail (around the origin).

where $B_0 = 1$ nT, $a = 2000$ km and the coordinates are defined as in Figure 2.

(a) Calculate the current density $\mathbf{j}(\mathbf{r})$ and the magnetic force density $\mathbf{j}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})$ (magnitudes and directions as functions of position). Also calculate their numerical values at $z = 0$. (3 p)

(b) How can such a current be sustained? Sketch possible orbits of ions and electrons in the y-z-plane – that is, not the plane of Figure 2, but a plane cutting it at right angles at $x = 0$, and tell why the trajectories look as you have drawn them, with reference to some relevant equation. (2 p)

(c) Now consider what happens if an instability appears in the region $-a < x < a$, $-10\,a < y < 10\,a$, $-a < z < a$ so that the resistivity in this region includes drastically. When the currents cannot flow through this region as before, where will they close now? Is this example relevant for any phenomenon in Earth's magnetosphere? (2 p)

Lycka till!

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for ”frozen-in” magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm’s law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_\perp \\ 0 \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \tfrac{ne}{B}\left(\tfrac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \tfrac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \tfrac{ne}{B}\left(\tfrac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \tfrac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\tfrac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \tfrac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2} m v_{\perp}^2 / B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu \nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v_F} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan \alpha = v_{\perp} / v_{\|}$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_{\mathrm{D}}}}{r}$$

Debye length:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_{\mathrm{p}} = \omega_{\mathrm{p}}/(2\pi) = \frac{1}{2\pi} \sqrt{\frac{ne^2}{\epsilon_0 m_{\mathrm{e}}}}$$

Rocket thrust:

$$T = v_{\mathrm{e}} \frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_{\mathrm{sp}} = \frac{\int T \, \mathrm{d}t}{m_{\mathrm{fuel}} g} = v_{\mathrm{e}}/g$$

The rocket equation:

$$\Delta v = -g t_{\mathrm{burn}} + v_{\mathrm{e}} \ln \left(1 + \frac{m_{\mathrm{fuel}}}{m_{\mathrm{payload+structure}}} \right)$$

Emitted thermal radiation power:

$$P_{\mathrm{e}} = \varepsilon \sigma A_{\mathrm{e}} T^4$$

Absorbed solar radiation power:

$$P_{\mathrm{a}} = \alpha A_{\mathrm{a}} I_{\mathrm{rad}}$$