Tentamen för Rymdfysik I och NV1
2005-03-11

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Institutionen för astronomi och rymdfysik
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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well. Answers may of course be given in Swedish or English, according to your own preference.

Time: 9:00 – 14:00

Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet, calculator. A bilingual dictionary, for example English-Swedish or English-French, may also be used.

1. Here follows a set of multiple choice questions, where you must find out which statements are correct. For each question (1-1, 1-2 etc), there is only one correct combination of answers, say "A and B" or "none". To score on a question, you need to have exactly the right combination. Any number of alternatives can be correct (0 – 3). (1 p/question, 10 p in total)

1:1. Which statements about sunspots are correct?

A. The number of sunspots varies with an 11 year period.
B. Sunspots are cooler than the surrounding plasma.
C. The magnetic field in a sunspot is stronger than in the surrounding plasma.

1:2. Which statements about the geostationary satellite orbit are correct?

A. The geostationary orbit is popular for communication satellites.
B. The geostationary orbit has a radius approximately six times the radius of the Earth.
C. The geostationary orbit is possible due to a balance between the gravitational pull of the earth and the moon.

1:3. Which statements about magnetic fields are correct?

A. The magnetic field from a source in vaccuum decreases with distance at least as fast as $1/r^3$.
B. The magnetic pressure arises because of the random motion of magnetized particles.
C. A frozen-in magnetic field is always static and cannot change in time.

1:4. Which statements about the Earth's magnetosphere are correct?

A. The radiation belts (van Allen bets) contain trapped energetic protons and electrons, corotating with the Earth.
B. Before it is released, the energy driving a geomagnetic substorm is mainly stored as magnetic energy in the geomagnetic tail.
C. The magnetosphere mostly disappears at nighttime, as there is no ionization by radiation from the sun at night.

1:5. Which statements about the solar wind are correct?

A. The interplanetary magnetic field is stronger than what we would expect from a dipole field from the sun, because the solar wind carries with it frozen-in magnetic fields from the sun.

B. The solar wind blows through the solar system in a spiral pattern (as seen in an inertial frame).

C. Solar wind particles hitting the Earth's upper atmosphere is the most important reason for formation of the ionosphere.

1:6. Which statements about space weather are correct?

A. High energy protons from solar flares and other major eruptions on the sun can cause radiation problems on satellites around the Earth.

B. A solar flare can cause disruption of electric power systems (transformers, power lines) around 8 minutes later.

C. Meteorologists need to keep close track of the space weather, as geomagnetic storms usually cause bad weather on Earth.

1:7. Which statements about the Earth's ionosphere are correct?

A. The electron density in the ionosphere is determined by the solar wind intensity and the Earth's magnetic field strength.

B. Due to collissions between particles, the conductivity in the direction parallel to the magnetic field is lower in the ionosphere than in the magnetosphere.

C. Field-aligned currents can flow along the magnetic field from the magnetic tail into the ionosphere, flow perpendicular to $\mathbf{B}$ in the ionosphere, and then flow up along the magnetic field lines from a different part of the ionosphere.

1:8. Which statements about the motion of charged particles are correct?

A. The orbital magnetic moment of an electron moving in a magnetic field is conserved if the field varies only a little during an electron gyroperiod and inside an electron gyroradius.

B. Only the magnetic field can do work on a charged particle, so we can always neglect the gravitational field when considering the motion of protons and electrons around the Earth.

C. In a dipole-like magnetic field, there are three basic periodicities for motion of charged particles: the gyroperiod, the bounce period (between magnetic mirrors), and the period for drifiting a complete orbit around the dipole.

1:9. Which statements about other planets are correct?

A. Venus does not have any known magnetic field but a dense atmosphere, and thus has an ionosphere but no magnetosphere.

B. Jupiter's magnetosphere is very large, because the solar wind has higher speed at Jupiter than at Earth and the Jovian atmosphere is very dense.

C. Planets and moons that have an atmosphere and are reached by ionizing radiation from the sun must also have an ionosphere.

1:10. In a homogeneous collisionless plasma in a constant and homogeneous magnetic field $\mathbf{B}$, a current can be generated by:

A. A constant and homogeneous electric field perpendicular to $\mathbf{B}$.

B. A constant and homogeneous gravitational field perpendicular to $\mathbf{B}$.

C. A constant and homogeneous gravitational field parallel to $\mathbf{B}$.

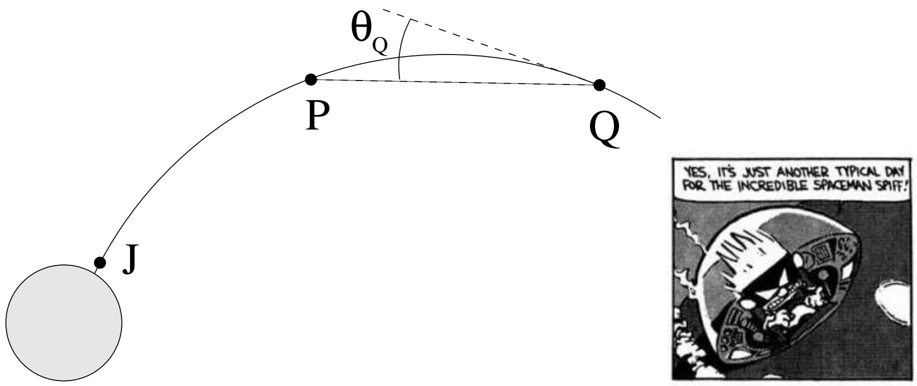

Figure 1: The tense situation when Spiff encounters the space monsters.

2. The bold spaceman Spiff, famous interplanetary explorer, cruises with his spaceship at a point P in the magnetosphere of the unexplored planet Zondarglash-B (see Figure 1). Evil space monsters in another space ship at a point Q at the same magnetic field line as P try to kill Spiff by blasting him with a deadly ray of ionized antimatter. The monsters, who did never pass their course in space physics, act intuitively and fire along the line of sight from Q to P. The ion gyroradius can be considered small compared to the scale length of inhomogeneities in the magnetic field. The magnetic field strength increases monotonically from Q down to J (the planetary ionosphere). The fight may end in three ways:

   (a) Triumph of the evil: Spiff is killed by the ion ray, monsters survive.

   (b) General doomsday: Spiff as well as the monsters are killed.

   (c) Justice prevails: The monsters are fried by the ion ray, while our hero Spiff survives to continue his glorious career.

   Explain why these cases arise. Deduce inequalities the angle θ_Q must satisfy for each of these cases. In the drama described above, the actual magnetic field values were $B_Q = 1$ μT, $B_P = 9$ μT, $B_J = 100$ μT, and $\theta_Q = 30°$. What was the outcome of the ferocious battle? (4 p)

3. Last night (2005-03-10 22:10 CET), the web site http://www.spaceweather.com listed the following current space weather conditions:

   - Solar wind speed: 602.0 km/s
   - Solar wind density: 0.4 protons/cm^{-3}
   - Interplanetary magnetic field:
     - B_{total}: 3.4 nT
     - B_z: 0.6 nT south

   (a) How would you say the solar wind values of yesterday night compare to what may be considered typical? (1 p)

   (b) Given these solar wind conditions, estimate the distance from the centre of the Earth to the magnetopause on the dayside. (2 p)

   (c) For the interplanetary magnetic field, they provide not only the total intensity but also the out-of-ecliptic (northward or southward) component. Why is this component more interesting for space weather than any of the two other vector components (dawn-dusk and sunward-antisunward)? (2 p)

   The Earth's magnetic field may be approximated by a dipole field with the strength 30 μT on the ground at the equator.

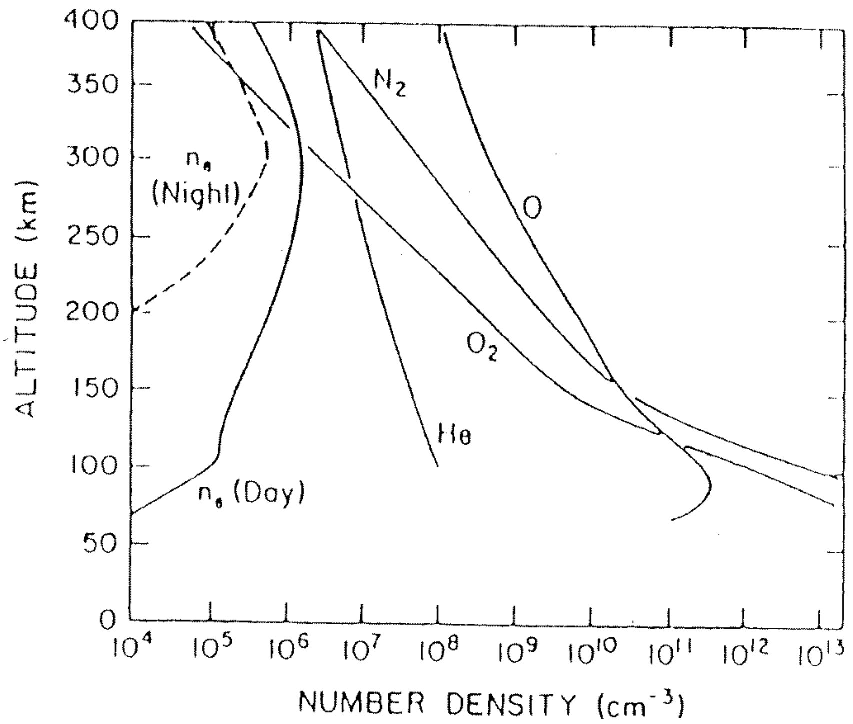

Figure 2: Altitude profiles of some neutral gas components, and of the electron density.

4. The diagram in Figure 2 shows the altitude distribution of some neutral atmospheric constituents, and the day and night profiles of the electron density in the Earth's ionosphere.

   (a) Derive (from the equation of motion of a neutral gas and an assumption of constant gravitational field) an expression showing why the concentrations of neutral molecules decrease approximately exponentially with increasing altitude, and why the concentration of atomic oxygen (O) decreases slower with altitude than the N_2 density, which in turn decreases slower than the concentration of molecular oxygen (O_2). State explicitly all assumptions you make. (2 p)

   (b) Why are the day- and nighttime profiles for the electron density different? (1 p)

5. The European Space Agency's Rosetta spacecraft was launched a year ago for catching up with a comet in the year 2014. Last Friday (March 4), Rosetta passed close (1900 km above the ground) to the Earth again, quite close to the equatorial plane. No thrusters (rockets) were fired during the flyby. The trajectory of Rosetta close to the Earth is shown in Figure 3. The coordinates are explained in the figure text.

   (a) Sketch the following in the figure (or in an accurate copy of it you make yourself): (2 p)

      A. The magnetopause
      B. The bow shock
      C. The plasmasphere
      D. Some solar wind stream lines (i.e. trajectories of some pracels of solar wind, with direction)

(b) If the spacecraft speed (in a frame moving with the Earth) was 3.9 km/s when Rosetta was far from the Earth, what was this speed when Rosetta was closest to Earth? (1 p)

(c) At the launch in March 2004, the total mass of the Ariane 5 rocket (including the 3 tons of Rosetta herself) was 749 tons, of which 637 tons were fuel. This was sufficient for giving Rosetta a final speed of some 3.5 km/s relative to the Earth, after having climbed out of the Earth's gravitational field. After the Earth flyby, the speed relative to the Earth has increased to 3.9 km/s. Estimate how much extra fuel the Ariane 5 rocket would have had to carry in order to give Rosetta these extra 0.4 km/s from the start, wothout any need for a flyby! Assume that the mass of the rocket itself would not have had to increase, only the fuel mass, that the rocket can be treated as a one-stage rocket with constant exhaust speed and that the fuel is burnt so fast that the burn time can be neglected. (In reality, none of these assumptions is true, and the real mass increase would have to have been bigger than your estimate) (3 p)

(d) We (the Swedish Institute of Space Physics in Uppsala) have provided one of the instruments onboard Rosetta. The sensors of this instrument are two spheres of titanium, 5 cm in diameter and covered with titanium nitride, for which the absorption and emission coefficients are 0.47 and 0.10, respectively. What temperature would you expect these sensors to attain during the Earth flyby, assuming they are in sunlight and that they do not exchange heat neither with the Earth nor with the spacecraft? There is no power dissipation within the spheres. (2 p)

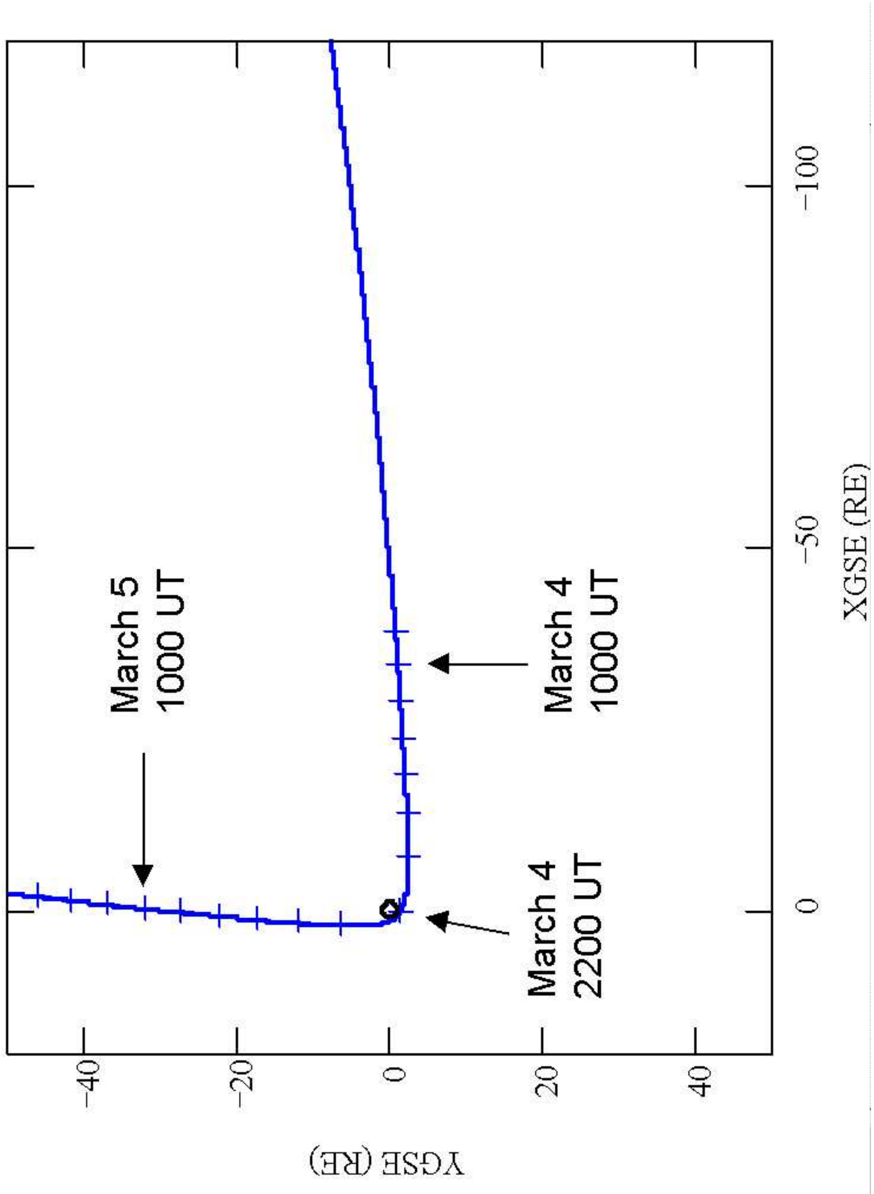

Figure 3: Rosetta trajectory during the first Earth flyby, March 4 – 5, 2005. The +X axis points toward the sun, the -Y axis is along the motion of the Earth around the sun, and the origin is in the centre of the Earth. Distances are given in units of Earth radii (6371.2 km).

Space Physics Formulas: Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_\perp \\ 0 \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \tfrac{ne}{B}\left(\tfrac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \tfrac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \tfrac{ne}{B}\left(\tfrac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \tfrac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\tfrac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \tfrac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2} m v_\perp^2 / B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu \nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v}_\mathbf{F} = \frac{\mathbf{F} \times \mathbf{B}}{q B^2}$$

Pitch angle:

$$\tan \alpha = v_\perp / v_\parallel$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_\mathrm{D}}}{r}$$

Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\epsilon_0 K T}{n e^2}}$$

Plasma frequency:

$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = \frac{1}{2\pi} \sqrt{\frac{n e^2}{\epsilon_0 m_\mathrm{e}}}$$

Rocket thrust:

$$T = v_\mathrm{e} \frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_\mathrm{sp} = \frac{\int T \, \mathrm{d}t}{m_\mathrm{fuel} g} = v_\mathrm{e}/g$$

The rocket equation:

$$\Delta v = -g t_\mathrm{burn} + v_\mathrm{e} \ln \left(1 + \frac{m_\mathrm{fuel}}{m_\mathrm{payload+structure}} \right)$$

Emitted thermal radiation power:

$$P_\mathrm{e} = \varepsilon \sigma A_\mathrm{e} T^4$$

Absorbed solar radiation power:

$$P_\mathrm{a} = \alpha A_\mathrm{a} I_\mathrm{rad}$$