Tentamen för Rymdfysik I och NV1
2004-03-12

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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well. Answers may of course be given in Swedish or English, according to your own preference.

Time: 15:00 – 20:00

Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet, calculator. A bilangual dictionary, for example English-Swedish or French-English, may also be used.

1. Here follows a set of qualitative questions, each of which should be answered in perhaps 5–15 lines of text, possibly an equation or two and maybe a figure.
   (a) What is an ionosphere? (1 p)
   (b) Why doesn't a satellite fall down to the ground when its engines are turned off? (1 p)
   (c) What is the pitch angle? (1 p)
   (d) How can plasmas survive in the universe, for example the solar wind – why don't the ions and electrons immediately recombine to form a neutral gas? (1 p)
   (e) What is the solar cycle? How long is it? What happens to the space weather in space around Earth during this cycle? (2 p)
   (f) What is a frozen-in magnetic field? Under what conditions can this concept be used? (2 p)
   (g) Draw a big, clear and nice figure showing the following regions around the Earth:
      i. The solar wind
      ii. The bow shock
      iii. The magnetopause
      iv. The plasmasphere

      (2 p)

2. The total mass launched by a rocket can be written

$$M = m_\mathrm{p} + m_\mathrm{f} + m_\mathrm{s}$$

   where m_p is the payload (sw: nyttolast) that we actually want to put into orbit, m_f is the fuel and m_s is the structural mass, i.e. the mass of the rocket itself.
   (a) Why is it at all good to divide a rocket into several stages? Explain in words. (2 p)
   (b) Show that dividing the rocket into two stages gives an additional Δv

$$(\Delta v)_\mathrm{bonus} = v_\mathrm{e} \ln \frac{1 + \frac{m_{2\mathrm{f}}}{m_{2\mathrm{s}} + m_\mathrm{p}}}{1 + \frac{m_{2\mathrm{f}}}{m_{1\mathrm{s}} + m_{2\mathrm{s}} + m_\mathrm{p}}}$$

   as compared to using the same fuel and structure mass in one single stage. (3 p)

   We assume that the rockets burn very quickly, so that the term gt_burn in the rocket equation can be neglected.

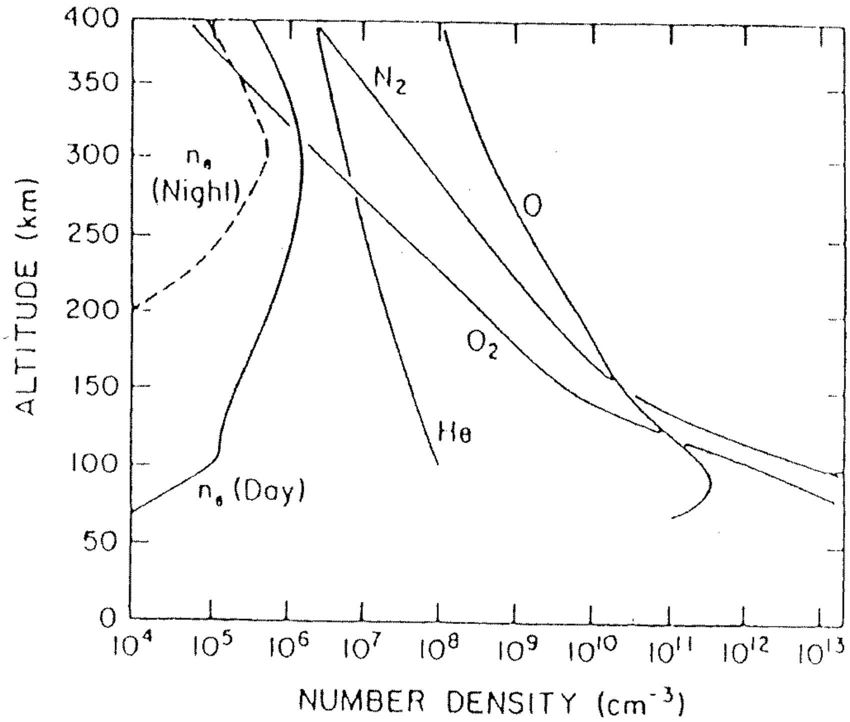

Figure 1: Altitude profiles of some neutral gas components, and of the electron density.

3. The diagram in Figure 1 shows the altitude distribution of some neutral atmospheric constituents, and the day and night profiles of the electron density in the Earth's ionosphere.
   (a) Derive (from the equation of motion of a neutral gas and an assumption of constant gravitational field) an expression showing why the concentrations of neutral molecules decrease approximately exponentially with increasing altitude, and why the concentration of atomic oxygen (O) decreases slower with altitude than the N_2 density, which in turn decreases slower than the concentration of molecular oxygen (O_2). State explicitly all assumptions you make. (2 p)
   (b) Why are the day- and nighttime profiles for the electron density different? (1 p)

4. (a) Show that the kinetic energy of a charged particle moving in a magnetic field, which is constant in time but may vary in space, is constant. (2 p)
   (b) A charged particle moving in a dipole field generally has three characteristic periods of the motion (or three characteristic frequencies, if you so prefer). Which are they? Here you only have to explain their physical meaning, give their usual names and tell why they exist, all in words, not to give any mathematical derivations. (2 p)
   (c) Consider an oxygen ion (O^+) with a kinetic energy of 10 keV and no velocity along the magnetic field, moving in the equatorial plane at a distance of 3 R_E from the center of the Earth. Calculate the two characteristic frequencies in (b) above which are defined for this particle (the third one is undefined because of the particle's zero velocity along the magnetic field). The geomagnetic field may be taken to be a dipole field with strength 30 μT on the ground at the equator. (3 p)

5. You are given the task to design a cheap spacecraft to Mars. The spacecraft is a cylinder of radius 1 m and height 1 m. It will be spinning around its axis, and thus have solar panels mounted all around its mantle surface. For reasons of power efficiency, the axis of the cylinder must be kept orthogonal to the line between the spacecraft and the sun. To keep operational costs and power consumption down, the spacecraft is to be turned off during all the passage from Earth to Mars, without even any heaters running, but must still have its axis perpendicular to the line to sun so as not to cause any expensive maneouvres. Your only way to control the temperature is to paint the surfaces (the solar panels on the mantle areas as well as the circular ends) with a transparent paint, which for economical reasons has to be the same on all surfaces. The spacecraft carries some intricate laboratory equipment for analyzing the Martian atmosphere, and this equipment, and hence all the spacecraft, must be kept at temperatures between +7°C and +87°C. If α and ϵ are the absorbtion and emission coefficients of your transparent paint, what range of values of α/ϵ is allowed? Is there such a range at all, or is it impossible to construct the spacecraft according to this specification? Mars is 1.52 times further from the sun than the Earth is. (5 p)

Lycka till!

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_\perp \\ 0 \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2} m v_\perp^2 / B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu \nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v}_\mathbf{F} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan \alpha = v_\perp / v_\parallel$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_\mathrm{D}}}{r}$$

Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = \frac{1}{2\pi} \sqrt{\frac{ne^2}{\epsilon_0 m_\mathrm{e}}}$$

Rocket thrust:

$$T = v_\mathrm{e} \frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_\mathrm{sp} = \frac{\int T \, \mathrm{d}t}{m_\mathrm{fuel} g} = v_\mathrm{e}/g$$

The rocket equation:

$$\Delta v = -g t_\mathrm{burn} + v_\mathrm{e} \ln \left(1 + \frac{m_\mathrm{fuel}}{m_\mathrm{payload+structure}} \right)$$

Emitted thermal radiation power:

$$P_\mathrm{e} = \varepsilon \sigma A_\mathrm{e} T^4$$

Absorbed solar radiation power:

$$P_\mathrm{a} = \alpha A_\mathrm{a} I_\mathrm{rad}$$