Tentamen för Rymdfysik I,
Rymdfysik MN1 och Rymdfysik NV1
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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well. Answers may of course be given in Swedish or English, according to your own preference.

Time: 9:00 – 14:00

Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet.

1. Here follows a set of qualitative questions, each of which should be answered in perhaps 5–15 lines of text, possibly an equation or two and maybe a figure.
   (a) Why do charged particles with non-zero speed perform circular gyromotion if placed in a magnetic field? (1 p)
   (b) Why are structures in plasmas often elongated (swedish: utsträckta) along the magnetic field lines? (1 p)
   (c) What is the Debye length? Why is it of interest? (1 p)
   (d) What is the pitch angle? (1 p)
   (e) Why are there plasmas at all in the universe? How do they come about? How do they survive – why don't the ions and electrons immediately recombine? (2 p)
   (f) What is a coronal mass ejection? What is the solar cycle? Are these two concepts related in any way? How can the solar activity influence the space environment around the Earth? (2 p)
   (g) Figure 1 shows the solar wind flow in front of a planetary magnetosphere. The solid curve is the magnetopause. What is the dashed curve called? How is the solar wind flow different in front of and behind this boundary? Draw the continuation of the solar wind stream lines from the dashed boundary. (2 p)

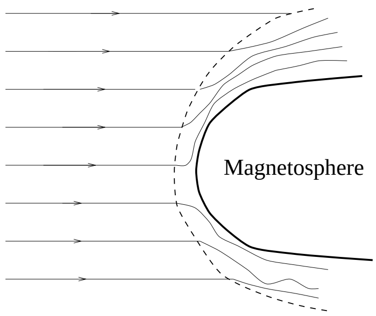

Figure 1: Illustration for problem 1g.

2. (a) Show that the kinetic energy of a charged particle moving in a magnetic field, which is constant in time but may vary in space, is constant. (2 p)
   (b) Consider an electron with a kinetic energy of 1 keV and no velocity along the magnetic field, moving in the equatorial plane at a distance of 4 R_E from the center of the Earth. How long time does it take for this electron to drift one complete orbit around the Earth? The geomagnetic field may be taken to be a dipole field with strength 30 μT on the ground at the equator. (3 p)

3. Consider a solar wind described by the velocity field

$$\mathbf{v}(\mathbf{r}) = v_0 \left(1 + \ln \frac{r}{R}\right) \hat{\mathbf{r}}, \qquad r \geq R$$

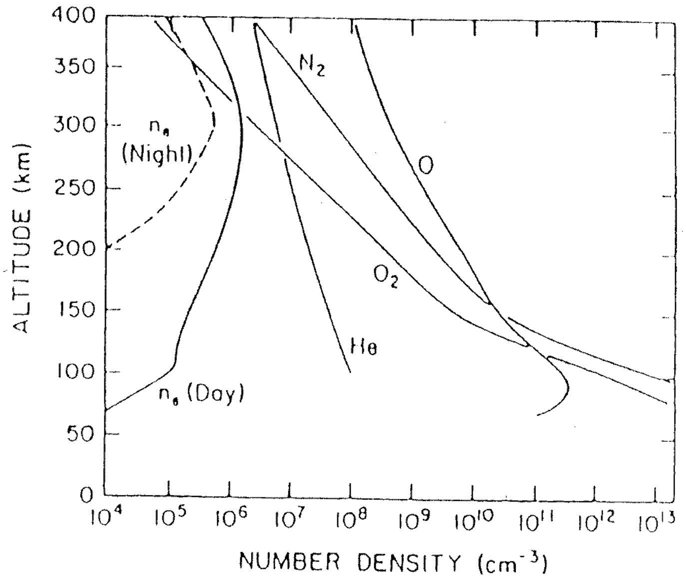

Figure 2: Altitude profiles of some neutral gas components, and of the electron density.

where r is the radial coordinate in a spherical coordinate system centred in the sun, and the constant v_0 is the solar wind speed at some distance $r = R$. Assume that the electric field in the rest frame of the solar wind is zero.

(a) Derive the plasma density profile $n(r)$ for $r \geq R$, given that $n(R) = n_0$. (2 p)

(b) If $\mathbf{B}(\mathbf{r}) = A\hat{\mathbf{r}} + C\hat{\phi}$, where A and C are constants, in the ecliptical plane ($\theta = 90°$) at $r = R$, derive expressions for B_r and B_ϕ valid for $r \geq R$ in the ecliptical plane. (3 p)

(c) Is the given solar wind velocity profile at all realistic? Motivate your answer. (1 p)

4. The diagram in Figure 2 shows the altitude distribution of some neutral atmospheric constituents, and the day and night profiles of the electron density in the Earth's ionosphere.

   (a) Derive (from the equation of motion of a neutral gas and an assumption of constant gravitational field) an expression showing why the concentrations of neutral molecules decrease approximately exponentially with increasing altitude, and why the concentration of atomic oxygen (O) decreases slower with altitude than the N_2 density, which in turn decreases slower than the concentration of molecular oxygen (O_2). State explicitly all assumptions you make. (2 p)

   (b) Why are the day- and nighttime profiles for the electron density different? (1 p)

5. The Rosetta spacecraft, Europe's comet chaser, is to go into space on an Ariane 5 rocket from French Guiana on January 13. The spacecraft itself may reasonably be described as a rectangular box of size 2 x 2 x 3 meters. All surfaces can be considered similar, with an emission coefficient $\epsilon = 0.3$. Rosetta is to meet up with a comet called Wirtanen close to its aphelion (maximim distance from sun) around 5 AU. The typical attitude of the spacecraft will be that one of its 2 x 3 meter surfaces faces the sun. When reaching the comet, Rosetta will follow it along its orbit towards perihelion (point closest to sun) during two years, at distances to the comet as small as one kilometer. Rosetta will even place a lander on the surface of the comet!

   (a) If the equilibrium spacecraft temperature, without any internal heating, is 40°C when the spacecraft is close to Earth orbit, what will it be when meeting Wirtanen at 5 AU? (2 p)

   (b) Assuming there is no thermal insulation, how much internal heating power (in watts) would be necessary to heat the spacecraft up to 0°C when at 5 AU? (2 p)

   (c) Assume the comet nucleus (that is, the solid part of the comet – the much bigger spectacular thing you see in the night sky is just dust, gas and plasma) to be spherical with a radius of 500 m and a mass density of about half that of liquid water. If Rosetta is to stay in a stable circular orbit around the Wirtanen nucleus at 1 km distance from the surface, what will be the speed of the spacecraft in its motion around the comet, and how long will it take to cover a full orbit around the comet? (2 p)

Lycka till!

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for ”frozen-in” magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm’s law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_{||} \end{pmatrix} \begin{pmatrix} E_\perp \\ 0 \\ E_{||} \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_{||} &= ne^2\left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_\perp^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu\nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v_F} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_\perp / v_\parallel$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0}\frac{e^{-r/\lambda_\mathrm{D}}}{r}$$

Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_\mathrm{e}}}$$

Rocket thrust:

$$T = v_\mathrm{e}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_\mathrm{sp} = \frac{\int T\,\mathrm{d}t}{m_\mathrm{fuel}g} = v_\mathrm{e}/g$$

The rocket equation:

$$\Delta v = -gt_\mathrm{burn} + v_\mathrm{e}\ln\left(1 + \frac{m_\mathrm{fuel}}{m_\mathrm{payload+structure}}\right)$$

Emitted thermal radiation power:

$$P_\mathrm{e} = \varepsilon\sigma A_\mathrm{e} T^4$$

Absorbed solar radiation power:

$$P_\mathrm{a} = \alpha A_\mathrm{a} I_\mathrm{rad}$$