Tentamen för Rymdfysik I och Rymdfysik MN1
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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well. Answers may of course be given in Swedish or English, according to your own preference.

Time: 9:00 – 14:00

Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet.

1. Here follows a set of qualitative questions, each of which should be answered in perhaps 5–15 lines of text, possibly an equation or two and maybe a figure.
   (a) What makes plasma physics so much more complicated than the physics of a neutral gas? (1 p)
   (b) What is the meaning of the concept ”frozen-in magnetic field”, and in what circumstances is it appliccable? (1 p)
   (c) Name at least two planets in the solar system, except the Earth, that should have magnetospheres. (1 p)
   (d) You have probably heard that ”in space there are no sounds, as there is nothing for sound waves to propagate in”. Well, now when you know that ”empty space” is not empty, what do you think? Are there sounds in space? Intelligent discussion is more important here than exactly right or wrong. (1 p)
   (e) What is the solar corona? What is (in science in general and in this course in particular) regarded as the most important unsolved problem regarding the corona? (2 p)
   (f) An argument sometimes heard goes as this: ”It is impossible to gain energy without using any fuel by so-called gravity assist manouvres, in which a spacecraft flies by close to a planet. As gravity is a conservative force, the speed of the spacecraft is the same when going away from the planet as when coming in towards it. Thus gravity assist is impossible.” Still gravity assist obviously works, without any active propulsion from rocket engines or solar sails or anything, as shown by many interplanetary spacecraft. How? (2 p)
   (g) Draw a **large** (use a separate paper) and **clear** sketch of the Earth's magnetosphere, indicating:
      i. Representative geomagnetic field lines, with direction
      ii. Representative solar wind flow lines, with direction
      iii. The bow shock
      iv. The magnetopause
      v. The plasmasphere

      (2 p)

2. It would be very interesting to send a spacecraft to do measurements of the plasma and the electromagnetic fields in the solar corona, for instance for solving the problem I hope you have mentioned in the solution to Problem 1e. However, going so close to the sun poses significant technical problems, above all thermal problems. One of the ideas of how to keep cool is to build a conical spacecraft with the top of the cone toward the sun. Assuming no internal dissipation of energy, derive an expression for the equilibrium temperature as a function of cone top (half-)angle and distance to the sun for such a spacecraft. What top angles would be needed at a distance of 40 solar radii from the sun in order to get down to temperatures of 400°C and 50°C, respectively, if the surface properties are $\alpha = 0.56$ and $\epsilon = 0.37$ (corresponding to e.g. unpolished steel)? The total solar luminosity is $3.9 \cdot 10^{26}$ W, and the solar radius is $696\,000$ km. (4 p)

3. (a) Derive an expression for the electron number density n_e in an ionosphere as a function of altitude h above the ground assuming that:

   - the neutral gas has the same temperature and composition on all heights while its number density varies as $n_\mathrm{n}(h) = n_0 \exp(-h/H)$, where $H = KT/(mg)$, m is the mean molecular mass and n_0 is the atmospheric number density at the ground,
   - the intensity I of the ionizing radiation increases with altitude as determined by $\mathrm{d}I = \sigma n_\mathrm{n}(h) I(h)\,\mathrm{d}h$,
   - ionization and recombination balances each other, so that $a_\mathrm{i} n_\mathrm{n}(h) I(h) = a_\mathrm{r} n_\mathrm{e}^2(h)$.

   Here, σ, a_i and a_r are constants, K is Boltzmann's constant, and g is the acceleration of gravity (assumed constant with altitude). (4 p)

   (b) Derive an expression for the altitude of the maximum electron density in (a) only depending on the constants H, σ and n_0. (2 p)

4. Consider an auroral electron (energy in the 10 keV range) at some point above the auroral zone.

   (a) Show, for instance by using an adiabatic invariant, that the particle is moving on the surface of a magnetic flux tube (which means that you shall show that the total magnetic flux inside the particle gyroorbit is constant). (2 p)

   (b) Derive the condition on the magnetic field strengths (locally and down in the atmosphere) that must be satisfied for the particle to reach the atmosphere before it is mirrored. (2 p)

5. The total mass launched by a rocket can be written

$$M = m_\mathrm{p} + m_\mathrm{f} + m_\mathrm{s}$$

   where m_p is the payload that we actually want to put into orbit, m_f is the fuel and m_s is the structural mass, i.e. the mass of the rocket itself.

   (a) Why is it at all good to divide a rocket into several stages? Describe in words. (1 p)

   (b) Show that dividing the rocket into two stages gives an additional Δv

$$(\Delta v)_\mathrm{bonus} = v_\mathrm{e} \ln \frac{1 + \frac{m_{2\mathrm{f}}}{m_{2\mathrm{s}} + m_\mathrm{p}}}{1 + \frac{m_{2\mathrm{f}}}{m_{1\mathrm{s}} + m_{2\mathrm{s}} + m_\mathrm{p}}}$$

   as compared to using the same fuel and structure mass in one single stage. (3 p)

   (c) Consider a one-stage rocket launch with $m_\mathrm{p} = 0.02\,M$ and $m_\mathrm{s} = 0.1\,M$. For the same payload, how much does Δv increase by dividing the rocket into two identical stages? Assume that the ratio of fuel to structure mass is the same for each stage and also for the one-stage rocket you compare to. The answer should be given in per cent of the one-stage Δv. (2 p)

   We assume that the rockets burn very quickly, so that the term gt_burn in the rocket equation can be neglected.

Lycka till!

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_\perp \\ 0 \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned}
\sigma_\mathrm{P} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\
\sigma_\mathrm{H} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\
\sigma_\parallel &= ne^2\left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right)
\end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_\perp^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu \nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v}_\mathbf{F} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_\perp / v_\parallel$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_\mathrm{D}}}{r}$$

Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_\mathrm{e}}}$$

Rocket thrust:

$$T = v_\mathrm{e} \frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_\mathrm{sp} = \frac{\int T \,\mathrm{d}t}{m_\mathrm{fuel} g} = v_\mathrm{e}/g$$

The rocket equation:

$$\Delta v = -g t_\mathrm{burn} + v_\mathrm{e} \ln\left(1 + \frac{m_\mathrm{fuel}}{m_\mathrm{payload+structure}}\right)$$

Emitted thermal radiation power:

$$P_\mathrm{e} = \varepsilon \sigma A_\mathrm{e} T^4$$

Absorbed solar radiation power:

$$P_\mathrm{a} = \alpha A_\mathrm{a} I_\mathrm{rad}$$