Tentamen för Rymdfysik I och Rymdfysik MN1
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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well.
Time: 14:00 – 19:00
Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet.

1. Here follows a set of qualitative questions, each of which should be answered in perhaps 5–15 lines of text, possibly an equation or two and maybe a figure.
   (a) What is gyromotion? (1 p)
   (b) What is the solar wind? (1 p)
   (c) The solar wind plasma has a temperature of typically 10 eV, i.e. around 100 000°C – so why doesn't an interplanetary spacecraft immediately melt or burn to ashes? (1 p)
   (d) What is the meaning of the concept ”frozen-in magnetic field”, and in what circumstances is it appliccable? (1 p)
   (e) Draw a graph of the electron number density n_{e} in the Earth's ionosphere as a function of altitude h from ground level up to the topside ionosphere. Explain why $n_{\mathrm{e}}(h)$ looks like you have drawn. (The absolute numerical values of n_{e} are not so important, but you should have reasonable values on the h axis.) (2 p)
   (f) Which of the following bodies can be expected to have an ionosphere:
      i. Venus
      ii. the Moon
      iii. the Milky Way galaxy
      iv. the Wind satellite (orbiting the Earth well outside Earth's magnetosphere)

      Motivate your answer in each case. It is the way you think and reason that is important – a well motivated but wrong answer may be accepted, while a short ”yes” or ”no” answer certainly will not. (2 p)
   (g) Draw a **large** (use a separate paper) and **clear** sketch of the Earth's magnetosphere, indicating:
      i. Representative geomagnetic field lines, with direction
      ii. Representative solar wind flow lines, with direction
      iii. The bow shock
      iv. The magnetopause
      v. The Van Allen radiation belts

      (2 p)

2. The Cluster spacecraft have recently detected electrons with energies between 100 eV and 1 keV just inside the frontlobe magnetopause of Earth at a distance of about 12 Earth radii from Earth's center. The electrons were observed when the spacecraft were about 30° above the (magnetic) equatorial plane ($\theta = 60°$).
   (a) Adopt a dipole field approximation and calculate the magnetic field strength at the Cluster location. The magnetic field strength at the surface of Earth near the equator is about 30 μT. (1 p)
   (b) The electrons are found only for pitch angles within $\pm 10°$ around the geomagnetic field direction and are counterstreaming (i.e. streaming in both directions with respect to the geomagnetic field direction). At what magnetic field strengths will these electrons mirror? At what altitudes above the Earth surface does the mirroring take place? The radius of the Earth is 6370 km. (3 p)

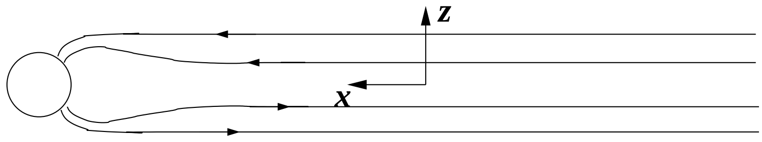

Figure 1: Idealized geometry of the relevant part of the geomagnetic tail.

3. Consider the following model of the magnetic field in the central part of the geomagnetic tail:

$$\mathbf{B}(\mathbf{r}) = \begin{cases} -B_0\hat{\mathbf{x}} & , \quad z < -a \\ B_0\,\hat{\mathbf{x}}\,\frac{3\,a^2\,z - z^3}{2\,a^3} & , \quad -a \leq z \leq a \\ B_0\hat{\mathbf{x}} & , \quad z > a \end{cases}$$

where $B_0 = 1$ nT, $a = 2000$ km and the coordinates are defined as in Figure 1.

(a) Calculate the current density $\mathbf{j}(\mathbf{r})$ and the magnetic force density $\mathbf{j}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})$ (magnitudes and directions as functions of position). Also calculate their numerical values at $z = 0$. (3 p)

(b) Now consider what happens if an instability appears in the region $-a < x < a$, $-10\,a < y < 10\,a$, $-a < z < a$ so that the resistivity in this region includes drastically. Instead of flowing through this region as before, the current now instead closes through field aligned currents and the ionosphere. How strong (in units of ampères) will these field-aligned currents be? Is this example relevant for any phenomenon in Earth's magnetosphere? (3 p)

4. Each of the four Cluster spacecraft, orbiting the Earth with apogee close to 20 Earth radii, is approximately a cylinder of radius 1.5 m and height 1 m, with the symmetry axis perpendicular to the direction of the sun. The mantle areas are covered with solar panels, while the top and bottom sides are mainly covered with a thermal blanket. Estimate the equilibrium temperature (in °C) of the satellites, assuming perfect thermal conductivity within the spacecraft and using data from Table 1. Also assume all onboard electrical systems are turned off. (3 p)

	Absorption coefficient	Emission coefficient
Solar panels	0.80	0.90
Thermal blanket	0.20	0.90

Table 1: Some thermal material properties.

5. (a) When launching a spacecraft into circular orbit at altitude h above the ground, the rocket must do work to increase the gravitational potential energy by some amount ΔU as well as to acquire the kinetic energy K corresponding to this orbit. Derive an expression for the ratio $\Delta U/K$ as a function of h. In what circumstances can one neglect either ΔU or K? (3 p)

(b) Providing a spacecraft with a magnetosphere of its own, by using an onboard electromagnet (possibly superconducting), is a suggested method for solar wind sailing. Estimate the dipole moment (in units of A·m^2) needed to provide a force balancing the solar gravitation at Earth orbit for a 20 kg microsatellite. Typical parameters for the solar wind can be taken to be $n_e = 5$ cm^{-3} and $v = 400$ km/s. (3 p)

(c) Is the result in (b) important in practice? Is it necessary that $F_{\text{sail}} > F_g$ if one wants to use this technique to travel outward through the solar system? Explain your answer. (1 p)

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_\perp \\ 0 \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_\perp^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu\nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v_F} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_\perp / v_\parallel$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_\mathrm{D}}}{r}$$

Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_\mathrm{e}}}$$

Rocket thrust:

$$T = v_\mathrm{e}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_\mathrm{sp} = \frac{\int T\,\mathrm{d}t}{m_\mathrm{fuel}g} = v_\mathrm{e}/g$$

The rocket equation:

$$\Delta v = -gt_\mathrm{burn} + v_\mathrm{e}\ln\left(1 + \frac{m_\mathrm{fuel}}{m_\mathrm{vehicle}}\right)$$

Emitted thermal radiation power:

$$P_\mathrm{e} = \varepsilon\sigma A_\mathrm{e} T^4$$

Absorbed solar radiation power:

$$P_\mathrm{a} = \alpha A_\mathrm{a} I_\mathrm{rad}$$