T 101018/1)

1:1 ABC

1:2 AB-

1:3 -B- Neutrons are neutral and cannot be trapped

1:4 A-- Power system disruptions can follow when a solar wind pressure pulse hits the Earth, which happens days after the flare, not minutes

1:5 -BC

1:6 A-C The magnetic field does no work on a charged particle, as $\bar{F} = q\bar{v} \times \bar{B} \perp \bar{v}$.

1:7 -B-

1:8 --C

1:9 --C

1:10 AB-

T 101018/2)

(a) The temperature is determined by a balance between the emitted radiation power

$$P_e = \varepsilon A_e \sigma T^4, \qquad (1)$$

where

ε = emission coefficient = 0.2

A_e = area emitting thermal radiation =
= total area = $2 \cdot \pi r^2 + 2\pi r \cdot h = 2\pi r(r+h)$

r = cylinder radius = 1 m

h = cylinder height = 1 m

σ = Stefan-Boltzmann's constant = $5.67 \cdot 10^{-8}$ W m^{-2}K^{-4}

T = equilibrium temperature,

and the absorbed solar power.

$$P_a = \alpha A_a I, \qquad (2)$$

where

α = the absorption coefficient we seek

A_a = absorbing area = area projected to sun =
$= \pi r^2$

I = solar intensity at 1 AU = 1370 W/m^2.

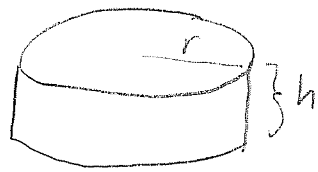
From $P_a = P_e$, we get

$$\alpha A_a I = \varepsilon A_e \sigma T^4 \qquad (3)$$

$$\alpha = \varepsilon \frac{A_e}{A_a} \cdot \frac{\sigma T^4}{I} = 0.2 \, \frac{2\pi \cdot 1 \cdot (1+1)}{\pi \cdot 1^2} \cdot \frac{5{,}67 \cdot 10^{-8} \cdot (273+27)^4}{1370} =$$

$$\approx 0.268$$

(b) A_e is unchanged as the full area is emitting, but A_a changes to



$$A_a' = 2r \cdot h = 2rh.$$

The absorbed power then becomes

$$P_a' = 2rh\alpha I. \qquad (4)$$

If the temperature should stay unchanged, we must then add an internal heating power P_i such that

$$P_a' + P_i = P_a \qquad (5)$$

$$\Rightarrow \quad P_i = P_a - P_a' = \alpha I (A_a - A_a') =$$

$$= \alpha I (\pi r^2 - 2rh) =$$

$$= 0.268 \cdot 1370 (\pi \cdot 1^2 - 2 \cdot 1 \cdot 1) \text{ W} =$$

$$= 0.268 \cdot 1370 (\pi - 2) \text{ W} \approx \underline{\underline{419 \text{ W}}}$$

T101018/3)

(a) The dominating pressure terms in the solar wind and in the magnetosphere are assumed to be the dynamic pressure p_D^{SW} and the magnetic pressure p_B^M, respectively. For the latter, we assume the geomagnetic field to be approximately dipolar with the subsolar point on the magnetopause approximately in the dipole equatorial plane. We thus take

$$p_D^{SW} \sim mnv^2,$$

where m is the typical particle mass, which we can take to be the proton mass for this estimate, and n and v the given solar wind density and speed.

Also,

$$p_B^M = \frac{B^2}{2\mu_0} \simeq \frac{B_0^2}{2\mu_0}\left(\frac{R_E}{r}\right)^6$$

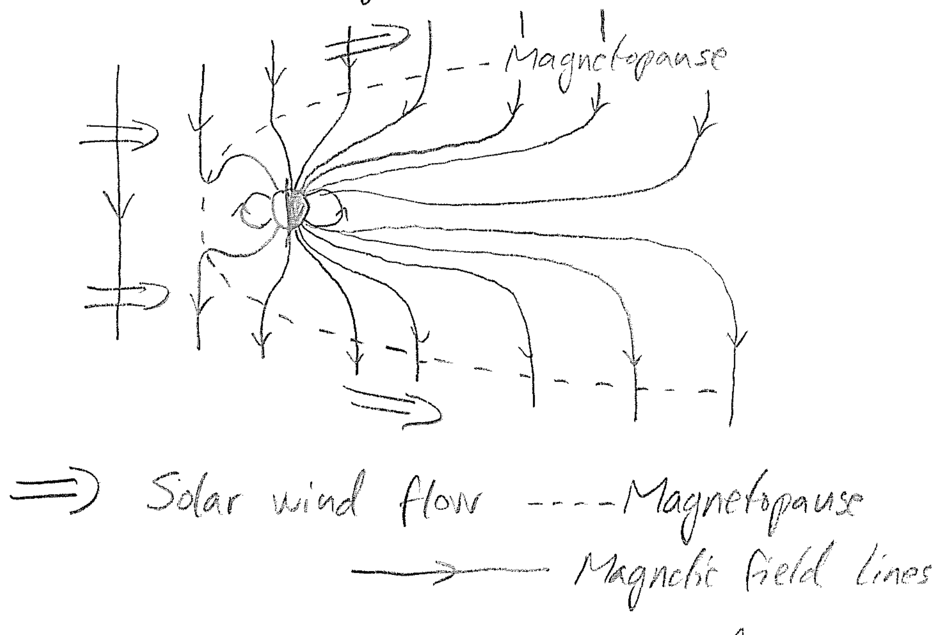
where $B_0 = 30\ \mu T$, $R_E = 6371.2$ km is the Earth radius, and r the distance we look for. We get

$$r \sim R_E\left[\frac{B_0^2}{2\mu_0 mnv^2}\right]^{1/6} =$$

$$= \left[\frac{(30\cdot 10^{-6})^2}{2\cdot 4\pi\cdot 10^{-7}\cdot 1{,}67\cdot 10^{-27}\cdot 6{,}4\cdot 10^6\cdot (290{,}4\cdot 10^3)^2}\right]^{1/6} R_E$$

$$\approx \underline{\underline{8.6\ R_E}}$$

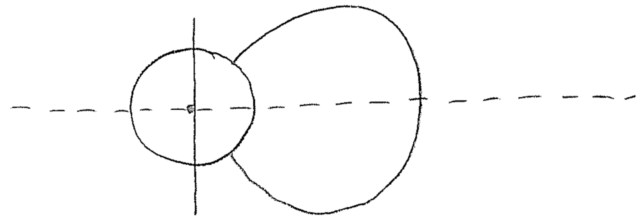
(b) IMF B_z is of particular interest as on the dayside magnetopause near the subsolar point, the geomagnetic field is more or less in the $\hat{z}$ direction. If IMF $B_z < 0$, i.e. for southward IMF, the fields can annihilate each other so that magnetic reconnection occurs. This opens for entry of solar wind energy and particles into the magnetosphere, and for storage of magnetic field energy in the magnetotail as the reconnected field lines are dragged into the tail by the solar wind flow.



The magnetic energy stored in the tail can be suddenly released in a geomagnetic substorm, which drives intensified field aligned currents and auroras. This explains the interest of IMF B_z for space weather.

T101018/4)

(a) The field line equation is given in the formula sheet as



$$\frac{r}{\sin^2\theta} = \text{const},$$

which we write as

$$r = r_0 \sin^2\theta .$$

$\theta = 90^\circ$ in the equatorial plane, so we immediately get

$$r_0 \sin^2 90^\circ = r_0 = 4\,R_E$$

as the field line is said to cut the equatorial plane at $4\,R_E$. Hence it cuts the Earth's surface ($r = R_E$) at a θ-value given by

$$\sin^2\theta = \frac{r}{r_0} = \frac{1}{4}$$

$$\Rightarrow \sin\theta = \pm\frac{1}{2}$$

$$\Rightarrow \theta = 30^\circ \text{ or } 150^\circ$$

This corresponds to a geomagnetic latitude

$$\lambda = 90^\circ - \theta = \underline{\underline{\pm 60^\circ}}$$

(b) As the ion has no parallel velocity (i.e. pitch angle $\alpha = 90^\circ$) out in the equatorial plane, where the magnetic field reaches its minimum value along the field line, it will always stay in this plane. The gyrofrequency is given by

$$f_c = \frac{1}{2\pi}\frac{qB}{m}, \tag{1}$$

so with

$$B = B_0\left(\frac{R_E}{r}\right)^3 \tag{2}$$

for a dipole field at $\theta = 90^\circ$, we get

$$f_c = \frac{1}{2\pi}\frac{qB_0}{m}\left(\frac{R_E}{r}\right)^3 = \tag{3}$$

$$= \frac{1}{2\pi}\frac{1{,}6\cdot 10^{-19}\cdot 30\cdot 10^{-6}}{16\cdot 1{,}67\cdot 10^{-27}}\left(\frac{1}{4}\right)^3 \text{ Hz} \approx \underline{\underline{0.47 \text{ Hz}}}$$

The gyroradius r_g is given by the general relation for circular motion,

$$V = r_g\,\omega_c = 2\pi r_g f_c \tag{4}$$

where V is the velocity, which we get from the energy $E = 100$ keV by

$$E = \frac{1}{2}mV^2. \tag{5}$$

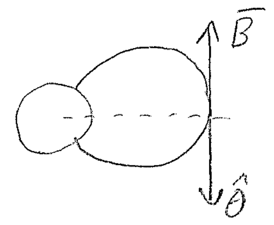
Combining gives

$$r_g = \frac{V}{2\pi f_c} = \frac{1}{2\pi f_c}\sqrt{\frac{2E}{m}} = \frac{1}{2\pi\cdot 0.47}\sqrt{\frac{2\cdot 100\cdot 10^3\cdot 1{,}6\cdot 10^{-19}}{16\cdot 1{,}67\cdot 10^{-27}}} \text{ m}$$

$$\approx \underline{\underline{371 \text{ km}}} \tag{6}$$

(c) We combine the force drift relation

$$\bar{v}_F = \frac{\bar{F} \times \bar{B}}{q B^2}$$



with the force on a magnetic dipole,

$$\bar{F} = -\mu \nabla B,$$

to get the ∇B drift,

$$\bar{v}_{\nabla B} = -\frac{\mu \nabla B \times \bar{B}}{q B^2} = \frac{\mu \bar{B} \times \nabla B}{q B^2}.$$

For a dipole field, B has its minimum with respect to θ in the equatorial plane, so $\partial B / \partial \theta = 0$ there. Hence, in the equatorial plane,

$$\nabla B = \hat{r} \frac{\partial B}{\partial r} = \hat{r} B_0 \frac{\partial}{\partial r} \left(\frac{R_E}{r} \right)^3 = -\frac{3}{r} B \hat{r},$$

where we used that

$$B = B_0 \left(\frac{R_E}{r} \right)^3$$

for a dipole field in the equatorial plane, where we also have that $\bar{B} = -B\hat{\theta}$. We thus get

$$\bar{v}_{\nabla B} = -\frac{3}{r} \cdot \frac{\mu (-B\hat{\theta}) \times B\hat{r}}{q B^2} = \frac{3\mu}{rq} \hat{\theta} \times \hat{r} = -\frac{3\mu}{rq} \hat{\varphi},$$

where the magnetic moment

$$\mu = \frac{\frac{1}{2} m v_\perp^2}{B} = \frac{\frac{1}{2} m v_\perp^2}{B_0} \left(\frac{r}{R_E} \right)^3.$$

Hence,

$$\bar{v}_{\nabla B} = -\frac{3}{rq} \frac{\frac{1}{2} m v_\perp^2}{B_0} \left(\frac{r}{R_E} \right)^3 \hat{\varphi} = -\frac{3}{R_E q} \frac{\frac{1}{2} m v_\perp^2}{B_0} \left(\frac{r}{R_E} \right)^2 \hat{\varphi} =$$

$$= -\frac{3 \cdot 100 \cdot 10^3 \cdot 1{,}6 \cdot 10^{-19}}{6371{,}2 \cdot 10^3 \cdot 1{,}6 \cdot 10^{-19} \cdot 30 \cdot 10^{-6}} \left(\frac{4}{1} \right)^2 \hat{\varphi} \text{ m/s} \approx$$

$$\approx \underline{-25 \hat{\varphi} \text{ km/s}}$$

The ion thus drifts westward with a speed of 25 km/s.

T 10/10/18/5)

(a) The plasma can flow quite easily along magnetic field lines, but not across it due to the gyration. Hence, plasma structures are usually elongated along the magnetic field, with steeper gradients of temperature, density etc across $\bar{B}$ than along it. The structures we see in the picture thus trace out magnetic field lines.

(b) The inclination is equal to the maximum latitude reached by the satellite, which we can estimate to be about $38°$ from the ground track figure.

(c) Looking at the thick curve in the figure, it is apparent that the ground track drifts $\sim 20°$ degrees westward in one orbit. This is due to the Earth's rotation, which is $360°$ in 24 hours, i.e. $15°/h$. To get a better estimate, we find that during 13 orbits, the point where the orbit cuts the equator on the northward track moves from $\sim 155°W$ to $\sim 165°E$. This means $\sim 320°$ in 13 orbits, or $\approx 25°/\text{orbit}$. With $15°/h$, $25°$ is about 1.67 h, which we take as an estimate of the period T. The period is related to orbital velocity v and orbital radius r by

$$V = \frac{2\pi r}{T}.$$

For a circular orbit, the acceleration is

$$a = \frac{v^2}{r}$$

which must be provided by gravitation, so that

$$\frac{v^2}{r} = \frac{GM}{r^2}.$$

Substituting v from above, we get

$$\frac{4\pi^2 r^2}{T^2} = \frac{GM}{r}$$

$\Rightarrow$

$$r^3 = \frac{GMT^2}{4\pi^2}$$

$\Rightarrow$

$$r = (GM)^{1/3}\left(\frac{T}{2\pi}\right)^{2/3} =$$

$$= (6{,}67 \cdot 10^{-11} \cdot 6 \cdot 10^{24})^{1/3} \cdot \left(\frac{1{,}67 \cdot 3600}{2\pi}\right)^{2/3} \text{ m} \approx$$

$$\approx 7155.7 \text{ km}$$

We thus get an altidue estimate as

$$h = r - R_E = (7155.7 - 6371.2) \text{ km}$$

$$\approx \underline{\underline{785 \text{ km}}}$$

[Error estimate: $\frac{\Delta r}{r} = \frac{2}{3}\frac{\Delta T}{T}$. We cannot be more than 1° per orbit wrong in our drift estimate (13° in 13 orbits), so $\frac{\Delta T}{T} \approx \frac{1}{25} \approx 0.04$, so $\frac{\Delta r}{r} \approx 0.03$. Thus we should have $h = 785 \pm 25$ km]