T 090318/1)

1:1) -B- (sunspots occur on the solar surface and can exist for months)

1:2) --C (solar wind recombination is very slow due to scarcity of particles; it would be impossible to create two tails with different charge)

1:3) AB-

1:4) --C (the rad belts do not corotate)

1:5) A-C (alternative B is plain nonsense)

1:6) --- (n_e in the ionosphere is determined by atmospheric density an ionizing radiation, mainly solar EUV, from space; auroras occur mainly above 100 km)

1:7) --C (A is nonsense; B does not look at the relevant reference frame which is the CoM system of the solar system)

1:8) ABC

1:9) ~~--C~~ -B-

1:10) -B- (IMF and solar wind density cannot influence what goes on on the Sun, only be influenced by this)

T 090318/2)

The absorbed solar radiation power is

$$P_a = \alpha A_a I, \qquad (1)$$

where α is given as 0.45, A_a is the absorbing area, and the solar intensity at heliocentric distance r is

$$I = I_0 \left(\frac{R}{r}\right)^2 \qquad (2)$$

with $R = 1$ AU and $I_0 = 1370$ W/m². The emitted power is

$$P_e = \varepsilon A_e \sigma T^4, \qquad (3)$$

where ε is given as 0.75, the emissive area

$$A_e = 2 A_a \qquad (4)$$

for a sun-facing disk, $\sigma = 5.67 \cdot 10^{-8}$ W m^{-2}K^{-4} is the Stefan-Boltzmann constant and T is the temperature.

At equilibrium, $P_e = P_a$, which with (1)-(4) gives

$$T = \left[\frac{P_e}{\varepsilon A_e \sigma}\right]^{1/4} = \left[\frac{P_a}{2\varepsilon A_a \sigma}\right]^{1/4} = \left[\frac{\alpha A_a I_0}{2\varepsilon A_a \sigma}\left(\frac{R}{r}\right)^2\right]^{1/4}$$

$$= \left[\frac{I_0}{2\sigma} \cdot \frac{\alpha}{\varepsilon} \cdot \left(\frac{R}{r}\right)^2\right]^{1/4} \qquad (5)$$

Numerically, we get for Solar Orbiter ($r = 45\, r_s$, where the solar radius $r_s = 696\,000$ km)

$$T_{SO} = \left[\frac{1{,}37 \cdot 10^3}{2 \cdot 5{,}67 \cdot 10^{-8}} \cdot \frac{0{,}45}{0.75} \cdot \left(\frac{150 \cdot 10^6}{45 \cdot 696 \cdot 10^3}\right)^2\right]^{1/4} \text{K} \approx 638 \text{ K}$$

$$\approx \underline{365\,°C},$$

and for Solar Probe ($r = 10\, r_s$)

$$T_{SP} = \sqrt{\frac{45}{10}}\, T_{SO} \approx 1355 \text{ K} \approx \underline{1081\,°C}$$

as T scales as $1/\sqrt{r}$ according to (5).

T090318/3)

(a) We assume a dipolar geomagnetic field, with

$$B = B_0 \left(\frac{R_E}{r}\right)^3$$

in the equatorial plane, with $B_0 = 30\ \mu T$. We also assume that inside the magnetopause, the magnetic pressure dominates, so that

$$p_{in} = p_B = \frac{B^2}{2\mu_0} = \frac{B_0^2}{2\mu_0}\left(\frac{R_E}{r}\right)^6 ,$$

while the solar wind dynamic pressure dominates outside:

$$p_{out} = p_{dyn} \approx n m v^2 ,$$

where the solar wind density $n = 0.6\ cm^{-3}$ and speed $v = 384.6$ km/s according to the data. For the particle mass, we take this to be $m = m_p$.

The pressures balance when $p_{in} = p_{out}$, so that we get

$$r = R_E \left[\frac{B_0^2}{2\mu_0 p_{in}}\right]^{1/6} = R_E \left[\frac{B_0^2}{2\mu_0 n m v^2}\right]^{1/6} =$$

$$= \left[\frac{(30\cdot 10^{-6})^2}{2\cdot 4\pi\cdot 10^{-7}\cdot 0.6\cdot 10^6\cdot 1,67\cdot 10^{-27}\cdot (384.6\cdot 10^3)^2}\right]^{1/6} R_E$$

$$\approx 11.6\ R_E \approx \underline{\underline{12\ R_E}}$$

as an estimate of the sought distance.

(b) On large scales in space and time, the electric field in the rest frame of the solar wind plasma should be zero:

$$\bar{E}' = \bar{E} + \bar{v} \times \bar{B} = 0, \qquad (1)$$

where $\bar{E}$ is the E-field as measured by a spacecraft, $\bar{B}$ the magnetic field, and $\bar{v}$ the solar wind velocity in the s/c frame of reference. Assuming

$$\bar{v} = -v\hat{x}, \qquad (2)$$

we get from (1) and (2) that

$$E_y = -(\bar{v} \times \bar{B})_y = -(v_z B_x - v_x B_z) =$$

$$= v_x B_z = -v B_z \qquad (3)$$

From the data,

$v = 384.6$ km/s

$B_z = 0.9$ nT (positive as it is northward),

so we get

$$E_y = -384.6 \cdot 10^3 \cdot 0.9 \cdot 10^{-9} \text{ V/m} \approx \underline{\underline{-0.35 \text{ mV/m}}}$$

T090318/4)

(a) The shortest is the gyroperiod, the time for the charge to complete an orbit under the Lorentz force $\bar{F} = q\bar{v} \times \bar{B}$.

Charges can also move along $\bar{B}$, but as B increases when we go away from the equatorial plane along a field line, the particle can be bouncing between magnetic mirrors in each hemisphere. This motion is periodic at the bounce period.

Finally, charges drift due to the gradient drift (and, when outside the equatorial plane, also the curvature drift, which has not been discussed in the course). This drift is east- or westward depending on the sign of the charge, and leads to a motion around the Earth at the drift period, which is the longest of the three.

(b) As the ion has $v_\parallel = 0$, and the magnetic field on a certain field line is minimal at the (magnetic) equator, the ion will stay in the equatorial plane. But it will drift according to the ∇B drift,

$$\bar{v}_B = \frac{-\mu \nabla B \times \bar{B}}{q B^2} \qquad (1)$$

where $\bar{B}$ is the dipole field

$$\bar{B} = -B_0 \left(\frac{R_E}{r}\right)^3 (2\hat{r}\cos\theta + \hat{\theta}\sin\theta) \qquad (2)$$

$$B = |\bar{B}| = B_0 \left(\frac{R_E}{r}\right)^3 \sqrt{4\cos^2\theta + \sin^2\theta} \qquad (3)$$

In the equatorial plane, $\theta = 90°$, so we get

$$\bar{B} = -B_0 \left(\frac{R_E}{r}\right)^3 \hat{\theta} = -B\hat{\theta} \qquad (4)$$

Hence only the $\hat{r}$ component of the gradient is of any interest for the vector product in (1) (there is no $\hat{\varphi}$ component, as seen from (3)), which from (3) is

$$(\nabla B)_r = \frac{\partial B}{\partial r} = -B_0 \frac{3}{r}\left(\frac{R_E}{r}\right)^3 \sqrt{4\cos^2\theta + \sin^2\theta}$$

$$= -\frac{3}{r} B \qquad (5)$$

Inserting into (1), we get

$$\bar{v}_{\nabla B} = -\frac{\mu}{qB^2}\left(-\frac{3}{r}B\hat{r}\right)\times\left(-B\hat{\theta}\right) = -\frac{3\mu}{qr}\hat{\varphi}. \qquad (6)$$

We thus get a drift in the periodic $\hat{\varphi}$ direction, with frequency given by

$$\frac{|\bar{v}_{\nabla B}|}{2\pi r} = \frac{3\mu}{2\pi q r^2} = \frac{3K_\perp}{2\pi q r^2 B} = \frac{3K_\perp}{2\pi q r^2 B_0}\left(\frac{r}{R_E}\right)^3 = \frac{3K_\perp}{2\pi q R_E^2 B_0}\cdot\frac{r}{R_E} =$$

$$\frac{3\cdot 10\cdot 10^3\cdot 1{,}6\cdot 10^{-19}\cdot 3}{2\pi\cdot 1.6\cdot 10^{-19}\cdot (6371{,}2\cdot 10^3)^2\cdot 30\cdot 10^{-6}}\ \text{Hz} \approx \underline{\underline{12\ \mu\text{Hz}}}$$

(c) The electron follows the field line, and so crosses the equatorial plane where the field line does so. The field line is defined by being everywhere parallel to $\bar{B}$,

$$d\bar{l} \parallel \bar{B}, \tag{1}$$

where $d\bar{l} = \hat{r}\, dr + \hat{\theta}\, r\, d\theta + \hat{\varphi}\, r \sin\theta\, d\varphi$ is the line element along the field line. We thus get

$$\frac{dr}{B_r} = \frac{r\,d\theta}{B_\theta} \tag{2}$$

as there is no B_φ. With the dipole field

$$\bar{B} = -B_0 \left(\frac{R_E}{r}\right)^3 \left(2\hat{r}\cos\theta + \hat{\theta}\sin\theta\right) \tag{3}$$

we get

$$\frac{dr}{2\cos\theta} = \frac{r\,d\theta}{\sin\theta} \tag{4}$$

$$\frac{dr}{r} = 2\,\frac{\cos\theta}{\sin\theta}\,d\theta \tag{5}$$

Integrating gives

$$\ln r = 2\ln|\sin\theta| + C = \ln \sin^2\theta + C \tag{6}$$

or

$$r = r_0 \sin^2\theta \tag{7}$$

with a change of integration constant to $r_0 = e^C$. The field line reaching ground ($r = R_E$) at $\lambda = 60°$, i.e. $\theta = 30°$, thus has

$$r_0 = \frac{r}{\sin^2\theta} = \frac{R_E}{\sin^2 30°} = \underline{4\,R_E}, \tag{8}$$

which we from (7) find to be the radial distance where it crosses the equatorial plane ($\theta = 90°$).

The evolution of the pitch angle α is set by the conservation of kinetic energy $\frac{1}{2}mv^2$ (as the Lorentz force does no work) and of the first adiabatic invariant $\mu = \frac{1}{2}mv_\perp^2/B$. As $v_\perp = v\sin\alpha$, we thus get that

$$\mu = \frac{\frac{1}{2}mv^2\sin^2\alpha}{B}, \qquad (9)$$

so that $\sin^2\alpha/B$ is conserved. From (3),

$$B = |\bar{B}| = B_0\left(\frac{R_E}{r}\right)^3\sqrt{4\cos^2\theta + \sin^2\theta} = B_0\left(\frac{R_E}{r}\right)^3\sqrt{4-3\sin^2\theta}. \qquad (10)$$

For the point in the equatorial plane, $\theta = 90°$ and

$$B_{eq} = B_0\left(\frac{R_E}{4R_E}\right)^3 = \frac{B_0}{64} = \frac{30}{64}\,\mu T \approx 0.469\,\mu T \qquad (11)$$

For the point at 10,000 km, we have from (7) & (8) that

$$\sin^2\theta_{10000} = \frac{r}{r_0} = \frac{R_E + h}{4R_E} \qquad (12)$$

where $h = 10,000$ km. Using this in (10), we get

$$B_{10000} = B_0\left(\frac{R_E}{R_E+h}\right)^3\sqrt{4 - 3\frac{R_E+h}{4R_E}} =$$

$$= 30\left(\frac{6371,2}{16371,2}\right)^3\sqrt{4 - 3\frac{16371,2}{4\cdot 6371,2}}\,\mu T \approx 2.55\,\mu T.$$

Using the conservation of $\sin^2\alpha/B$ and $\alpha_{10000} = 90°$,

$$\frac{\sin^2\alpha_{10000}}{B_{10000}} = \frac{\sin^2\alpha_{eq}}{B_{eq}} \qquad (13)$$

and

$$\alpha_{eq} = \arcsin\left(\sqrt{\frac{B_{eq}}{B_{10000}}}\sin 90°\right) =$$

$$= \arcsin\left(\sqrt{\frac{0.469}{2.55}}\right) \approx \underline{\underline{25°}} \qquad (14)$$

T090318/5)

(a) The centripetal acceleration in circular motion is

$$a = \frac{v^2}{r}, \qquad (1)$$

and this must be provided by the gravitational force,

$$a = \frac{F}{m} = \frac{GM}{r^2}, \qquad (2)$$

where M is the Earth mass. The period of the geostationary orbit is $T = 24$ h as this is the rotation period of the Earth, and

$$v = \frac{2\pi r}{T}. \qquad (3)$$

Combining (1)-(3), we get

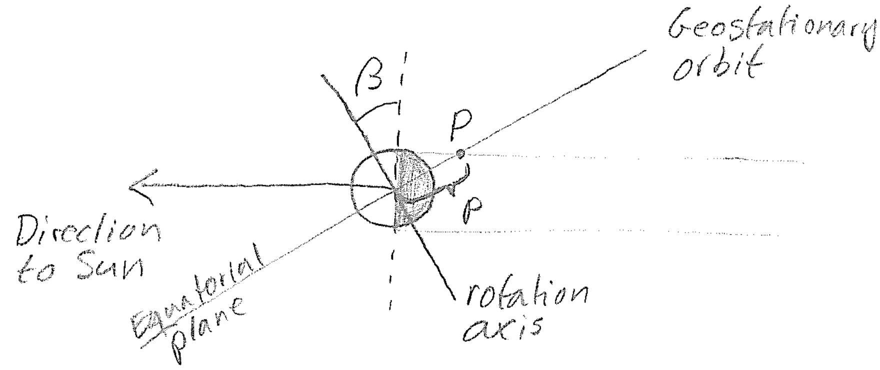
$$\frac{GM}{r^2} = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2} \qquad (4)$$

$$r^3 = \frac{GMT^2}{4\pi^2} \qquad (5)$$

$$r = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{6.67\cdot10^{-11}\cdot 6\cdot10^{24}\cdot(24\cdot3600)^2}{4\pi^2}} \text{ m}$$

$$\approx 42\,300 \text{ km} \approx \underline{\underline{6.64\ R_E}}$$

(b) If it does not happen on every orbit, that would be due to the tilt of the rotation axis of the Earth, $\beta = 23.45°$ (from the table). So we need only look at the extreme case: at solstices, when the geometry is as in the following sketch:



Let us take the shadow of the Earth to be a circular cone: in reality, it will be conical, due to the finite size of the Sun (and refraction effects in the terrestrial atmosphere), so if we get constant illumination in the case of cylindrical shadow, we will have it in reality as well.

With an axial tilt $\beta = 23.45°$, the point P where the shadow cylinder crosses the equatorial plane behind the Earth is at a geocentric distance

$$\rho = \frac{R_E}{\sin \beta} = \frac{R_E}{\sin 23.45°} \lesssim 2.52\ R_E$$

This means that there is ample margin to the geostationary orbit at $6.64\ R_E$, so clearly eclipses do not happen on every orbit