T080402/1)

1:1 ABC

1:2 B

1:3 A

1:4 AB

1:5 A

1:6 ABC

1:7 AC

1:8 AB

1:9 BC

1:10 AC

T080402/2)

Assuming there are no other heat sources or sinks, the temperature T of the sensor is determined by a balance

$$P_a = P_e \tag{1}$$

between absorbed solar radiation power P_a and emitted heat radiation power P_e. Here

$$P_a = \alpha I A_a \tag{2}$$

where $\alpha = 0.47$ is given and I is the solar energy flux at distance R from the sun, given by

$$I = I_0 \left(\frac{R_0}{R}\right)^2 \tag{3}$$

where $I_0 = 1370$ W/m^2 is the value of I at $R = R_0 = 1$ AU, and A_a is the effective absorbing area, i.e. the sensor area projected to the sun, which will be a circle of radius $a = 25$ mm, i.e.

$$A_a = \pi a^2. \tag{4}$$

For the emitted power,

$$P_e = \varepsilon \sigma T^4 A_e \tag{5}$$

where $\varepsilon = 0.10$ is given, σ is the Stefan-Boltzmann constant, and A_e the effective emitting area, which will be the total area of the sensor:

$$A_e = 4\pi a^2. \tag{6}$$

Combining (1)-(6), we get

$$\alpha I_0 \left(\frac{R_0}{R}\right)^2 \cdot \pi a^2 = \varepsilon \sigma T^4 \cdot 4\pi a^2 \tag{7}$$

$$\Rightarrow \quad T = \sqrt[4]{\frac{\alpha I_0}{4 \varepsilon \sigma} \left(\frac{R_0}{R}\right)^2}$$

The extreme temperature values will clearly be found at the extreme distances to the sun, i.e.

$$T_{max} = \left[\frac{0{,}47 \cdot 1370}{4 \cdot 0.1 \cdot 5{,}67 \cdot 10^{-8}} \left(\frac{1}{0.72}\right)^2\right]^{1/4} K \approx 484\ K \approx 211\ °C$$

T_{min} can be calculated the same way, or obtained by scaling T_{max}:

$$T_{min} = T_{max} \sqrt{\frac{0.72}{9.54}} \approx 133\ K \approx -140\ °C$$

Answer: $-140°C$ and $+211°C$

T080402/3)

(a) The force on the particle is

$$\bar{F} = q\,\bar{v} \times \bar{B}, \tag{1}$$

where q is the charge, $\bar{v}$ the particle velocity and $\bar{B}$ the magnetic field. By Newton's second law,

$$\bar{F} = m\frac{d\bar{v}}{dt}, \tag{2}$$

so we get the evolution of the kinetic energy as

$$\frac{d}{dt}\left(\frac{1}{2}mv^2\right) = \frac{d}{dt}\left(\frac{1}{2}m\bar{v}\cdot\bar{v}\right) = \bar{v}\cdot m\frac{d\bar{v}}{dt} =$$

$$= \bar{v}\cdot\bar{F} = q\,\bar{v}\cdot(\bar{v}\times\bar{B}) = 0. \tag{3}$$

Hence, the particle kinetic energy is conserved, QED.

(b) As the particle has $90°$ pitch angle in the equatorial plane, where the B field strength has its minimum, the particle will stay in this plane. The bounce frequency is thus undefined. The gyro frequency is given by

$$f_c = \frac{1}{2\pi}\frac{qB}{m} = \frac{1}{2\pi}\frac{eB}{m} \tag{4}$$

as $q = e$ for O^+. In our case,

$$B = B_0\left(\frac{R_E}{r}\right)^3, \tag{5}$$

so we get

$$f_c = \frac{1}{2\pi}\frac{eB_0}{m}\left(\frac{R_E}{r}\right)^3 = \frac{1.6\cdot10^{-19}\cdot30\cdot10^{-6}}{2\pi\cdot16\cdot1{,}67\cdot10^{-27}}\left(\frac{1}{3}\right)^3 \text{ Hz} \approx$$

$$\approx 1.1 \text{ Hz}, \tag{6}$$

where we used that $B_0 = 30\ \mu T$.

The third (or, in this case, the second) period of the motion is the drift period around the equatorial plane due to the gradient drift. The drift velocity is

$$\bar{v}_d = \frac{\bar{B} \times \mu \nabla B}{q B^2} \tag{7}$$

and the distance around the equator is $2\pi r$, so the drift frequency is

$$f_d = \frac{|\bar{v}_d|}{2\pi r}. \tag{8}$$

In the equatorial plane,

$$\bar{B} = -B_0 \left(\frac{R_E}{r}\right)^3 \hat{\theta} \tag{9}$$

and

$$B = B_0 \left(\frac{R_E}{r}\right)^3 \tag{10}$$

$$\Rightarrow \nabla B = -\frac{3}{r} B_0 \left(\frac{R_E}{r}\right)^3 \hat{r} = -\frac{3B}{r} \hat{r} \tag{11}$$

so that $\nabla B \perp \bar{B}$ and

$$v_d = |\bar{v}_d| = \frac{B \cdot \mu \frac{3B}{r}}{q B^2} = \frac{3\mu}{qr} \tag{12}$$

(where we used that $\frac{\partial B}{\partial \theta} = 0$ in the equatorial plane because of symmetry – this <u>cannot</u> be deduced from (10), which is a special case for $\theta = 90°$!). Now

$$\mu = \frac{E_\perp}{B} \tag{13}$$

where $E_\perp = 10$ keV is the particles energy due to its gyration motion (as the pitch angle was 90°, this is all the energy). Hence,

$$f_d = \frac{V_d}{2\pi r} = \frac{3\mu}{2\pi q r^2} = \frac{3E_\perp}{2\pi q r^2 B} = \frac{3E_\perp}{2\pi q r^2 B_0}\left(\frac{r}{R_E}\right)^3 =$$

$$= \frac{3E_\perp}{2\pi q R_E^2 B_0} \cdot \frac{r}{R_E} \qquad (14)$$

which gives

$$f_d = \frac{3 \cdot 10 \cdot 10^3 \cdot \cancel{1{,}6 \cdot 10^{-19}}}{2\pi \cdot 1{,}6 \cdot 10^{-19} \cdot (6{,}371 \cdot 10^6)^2 \cdot 30 \cdot 10^{-6}} \cdot 3 \quad Hz \approx$$

$$\approx 1.2 \cdot 10^{-5} \ Hz$$

(meaning that the drift period is about a day, 24 h)

T080402/4)

(a) The maximum latitude is equal to the inclination. From the diagram, we get $i \approx 63°$. (The sign of the inclination is given by the sense of rotation around the Earth: as Freja moves eastward, its inclination is positive).

(b) In a circular orbit, the centripetal acceleration is

$$a = \frac{v^2}{r}, \qquad (1)$$

so gravity much provide a force

$$ma = \frac{GmM}{r^2} \qquad (2)$$

for such an orbit to be possible. Now the period T and the speed v are related by

$$vT = 2\pi r \qquad (3)$$

so we get

$$\frac{v^2}{r} = \frac{GM}{r^2} \qquad (4)$$

$$r = \frac{GM}{v^2} = \frac{GMT^2}{4\pi^2 r^2} \qquad (5)$$

$$r^3 = \frac{GM}{4\pi^2} T^2 \qquad (6)$$

$$r = \sqrt[3]{\frac{GM}{4\pi^2} T^2} = \sqrt[3]{\frac{6.67 \cdot 10^{-11} \cdot 6 \cdot 10^{24}}{4\pi^2} \cdot (3600 + 50 \cdot 60)^2} \text{ m}$$

$$\approx 7615 \text{ km}$$

$$h = r - R_E = (7615 - 6371) \text{ km} \approx 1244 \text{ km}$$

$$v = \frac{2\pi r}{T} = \frac{2\pi \cdot 7615 \cdot 10^3}{3600 + 50 \cdot 60} \text{ m/s} \approx 7.25 \text{ km/s}$$

Answer: $v = 7.2$ km/s and $h = 1244$ km

(c) If the orbit is circular, the B-field strength in a dipole field only depends on the latitude, being stronger toward the poles, because

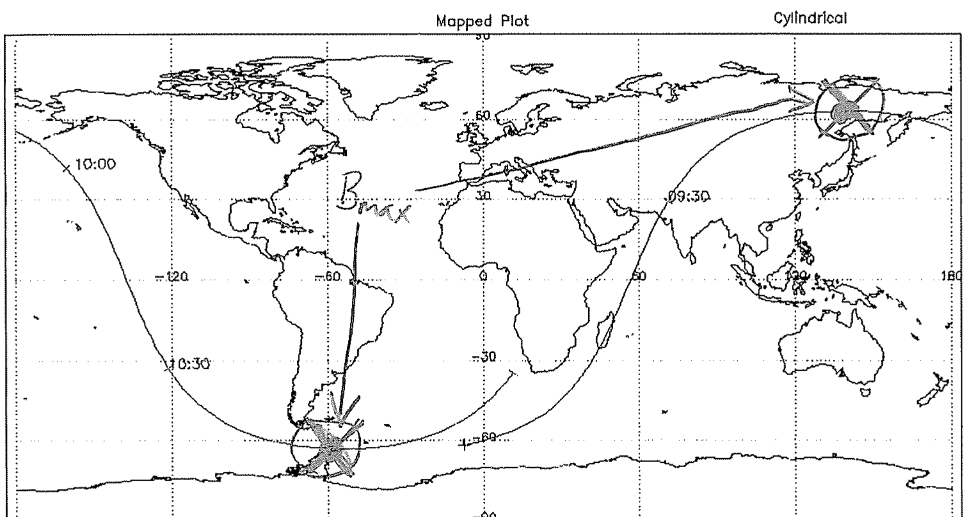
$$B = |\bar{B}| = B_0\left(\frac{R_E}{r}\right)^3 \sqrt{4\cos^2\theta + \sin^2\theta} =$$

$$= B_0\left(\frac{R_E}{r}\right)^3 \sqrt{1+3\cos^2\theta} = B_0\left(\frac{R_E}{r}\right)^3 \sqrt{1+3\sin^2\lambda} \quad (7)$$

where $\lambda = 90^\circ - \theta$ is the latitude. We thus get B_{max} at max latitude $\approx 63^\circ$,

$$B_{max} = B_0\left(\frac{R_E}{r}\right)^3 \sqrt{1+3\sin^2 63^\circ} =$$

$$= 30 \cdot \left(\frac{6371}{7615}\right)^3 \sqrt{1+3\sin^2 63^\circ}\ \mu T \approx 34\ \mu T$$



+ FREJA Radial Trace in Geographic Coordinates: Time Range 6/13/1995 (164) 9:0 6/13/1995 (164) 11:0

(d) If the plasma is co-rotating with the Earth, its velocity at a distance ρ from the rotation axis is

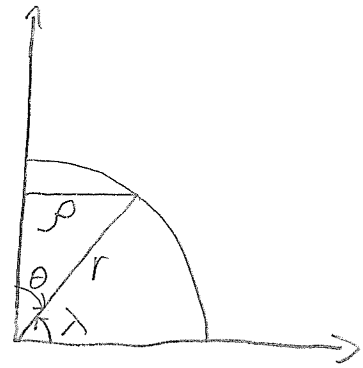
$$\bar{V}_p = \rho\Omega\hat{\varphi} \quad (8)$$

where Ω is the angular speed due to Earth's rotation,

$$\Omega = \frac{2\pi}{24 \cdot 3600} \text{ rad/s.}$$

We have

$$\rho = r \sin\theta, \qquad (9)$$



so that at the points of maximum B we get

$$\bar{V}_\rho = r \sin\theta \cdot \Omega\, \hat{\varphi} =$$

$$= 7615 \cdot \sin(90^\circ - 63^\circ) \cdot \frac{2\pi}{24 \cdot 3600}\, \hat{\varphi}\ \text{km/s} \approx 0.25\, \hat{\varphi}\ \text{km/s}. \qquad (10)$$

At these points in the orbit, the s/c velocity $\bar{V}_{sc}$ is purely in the $\hat{\varphi}$ (eastward) direction, so that

$$\bar{V}_{sc} = V \hat{\rho} = 7.25\, \hat{\varphi}\ \text{km/s} \qquad (11)$$

(using the result from (b)). In the reference frame of the s/c, the plasma thus moves at a velocity

$$\bar{V} = \bar{V}_\rho - \bar{V}_{sc} = -7\, \hat{\varphi}\ \text{km/s}. \qquad (12)$$

The electric field in the plasma rest frame should be zero, i.e.

$$0 = \bar{E} + \bar{V} \times \bar{B} \qquad (13)$$

where $\bar{E}$ is the E-field in the Freja frame. Hence,

$$\bar{E} = -\bar{V} \times \bar{B} \qquad (14)$$

0.123

where

$$\bar{B} = -B_0 \left(\frac{R_E}{r}\right)^3 (2\hat{r}\cos\theta + \hat{\theta}\sin\theta). \qquad (15)$$

Hence, in the northern hemisphere at $\lambda = 63^\circ$,

$$\bar{E} = -7 \cdot 10^3 \cdot 30 \cdot 10^{-6} \left(\frac{6371}{7615}\right)^3 \left(2\hat{\varphi} \times \hat{r} \cos[90^\circ - 63^\circ] + \hat{\varphi} \times \hat{\theta} \sin[90^\circ - 63^\circ]\right) \frac{V}{m}$$

$$\approx (-219\,\hat{\theta} + 56\,\hat{r})\ \text{mV/m} = (+56\,\hat{r} - 219\,\hat{\theta})\ \text{mV/m}$$

The $\hat{\theta}$ term changes sign in the south to give $\bar{E} = (+56\,\hat{r} + 219\,\hat{\theta})$ mV/m there.

T080402/5)

(a) Until around 14:00 in day 297 ($t = 297.63$ days), solar wind conditions are normal. A southward turn of the IMF around $t = 294.32$ days may have started loading of B-field energy into the magnetotail, possibly released in a few substorms, but otherwise nothing remarkable happens in this time interval.

At $t = 297.63$ days, an uncomononly strong solar wind intensification occurs. Density goes up to 70 cm^{-3} in the peaks, combined with large B and high v. This is obviously a major geomagnetic storm, leading to highly disturbed space weather.

(b) We estimate the standoff distance by finding where the magnetic pressure from the geomagnetic field,

$$p_B = \frac{B^2}{2\mu_0} = \frac{B_0^2}{2\mu_0}\left(\frac{R_E}{r}\right)^6$$

balances the solar wind dynamic pressure

$$p_{sw} \approx m_p n v^2,$$

which gives

$$r = R_E \left[\frac{B_0^2}{2\mu_0 m_p n v^2}\right]^{1/6}.$$

With values from the figure, we get

$$r(t = 297.5) \approx R_E \left[\frac{(30 \cdot 10^{-6})^2}{2 \cdot 4\pi \cdot 10^{-7} \cdot 1{,}67 \cdot 10^{-27} \cdot 5 \cdot 10^6 \cdot (440 \cdot 10^3)^2}\right]^{1/6} \approx 7.8\ R_E$$

$$r(t = 297.74) \approx R_E \left[\frac{(30 \cdot 10^{-6})^2}{2 \cdot 4\pi \cdot 10^{-7} \cdot 1{,}67 \cdot 10^{-27} \cdot 70 \cdot 10^6 \cdot (580 \cdot 10^3)^2}\right]^{1/6} \approx 4.6\ R_E$$

This last value is very low, meaning that the geostationary orbit reached outside the magnetopause!