T070309/1)

1. A
2. ABC
3. B
4. AB
5. -
6. BC
7. A
8. ABC
9. B
10. AC

T070309/2)

(a) For a pitch angle of 45°, the particle kinetic energy is equally partitioned between parallel and perpendicular. Hence, the initial kinetic energies were

$$K_\perp^{(0)} = \frac{20}{2}\,eV = \underline{\underline{10\,eV}} \qquad (1)$$

$$K_\parallel^{(0)} = K_\perp^{(0)} = \underline{\underline{10\,eV}}. \qquad (2)$$

The acceleration only affects $K_\parallel$, so we get

$$K_\perp^{(1)} = K_\perp^{(0)} = \underline{\underline{10\,eV}} \qquad (3)$$

$$K_\parallel^{(1)} = K_\parallel^{(0)} + 1\,keV = \underline{\underline{1.01\,keV}} \qquad (4)$$

because the electron falls through a potential drop of 1 kV.

(b) The pitch angle after acceleration is given by

$$\tan\alpha^{(1)} = \frac{v_\perp^{(1)}}{v_\parallel^{(1)}} = \sqrt{\frac{K_\perp^{(1)}}{K_\parallel^{(1)}}} = \sqrt{\frac{10}{1010}} \qquad (5)$$

$$\Rightarrow \alpha \approx 5.7^\circ \qquad (6)$$

After the potential drop, kinetic energy and orbital magnetic moment are consered:

$$K = K_\parallel + K_\perp = \text{constant} \qquad (7)$$

$$\mu = \frac{K_\perp}{B} = \frac{K\sin^2\alpha}{B} = \text{constant} \qquad (8)$$

$$\Rightarrow \frac{\sin^2\alpha}{B} = \text{constant} \qquad (9)$$

$$\Rightarrow \sin^2\alpha_i = \frac{B_i}{B^{(1)}}\sin^2\alpha^{(1)} \qquad (10)$$

where index i refers to values in the ionosphere.
With $B_i / B^{(1)} = 1/0.02 = 50$, we get

$$\sin^2 \alpha_i = 50 \sin^2 \alpha^{(1)} \approx 0.49. \tag{11}$$

As $\sin^2 \alpha_i < 1$, the particle is inside the loss cone and will be able to reach the ionosphere.

T070309/3)

(a) The quantity deciding the dynamics will be the quantity of highest energy density (w). For the magnetic field and bulk flow kinetic energy densities, we have

$$w_B = \frac{B^2}{2\mu_0}$$

$$w_K = \frac{1}{2}\rho_m v^2$$

where B is the magnetic field strength, ρ_m the mass density and v the flow speed. As protons dominate the solar wind,

$$\rho_m \approx n m_p .$$

From Figure 2, we have

$$1\ \text{cm}^{-3} \lesssim n \lesssim 10\ \text{cm}^{-3}$$

$$350\ \text{km/s} \lesssim v \lesssim 650\ \text{km/s}$$

$$3\ \text{nT} \lesssim B \lesssim 10\ \text{nT}$$

Thus we get

$$w_B \lesssim \frac{\overbrace{10^2 \cdot 10^{-18}}^{(10\,\text{nT})^2}}{2 \cdot 4\pi \cdot 10^{-7}}\ \text{J/m}^3 \approx 10^{-10}\ \text{J/m}^3$$

$$w_K \gtrsim \frac{1}{2} \underbrace{1 \cdot 10^6}_{\substack{1\,\text{cm}^{-3} \\ = 10^6\,\text{m}^{-3}}} \cdot 1{,}67 \cdot 10^{-27} \cdot 350^2 \cdot 10^6\ \text{J/m}^3 \approx \approx 10^{-10}\ \text{J/m}^3$$

From Figure 2, we can note that the times of maximum B is not where n and v are low, so we will have $w_K > w_B$, so the bulk flow dominates the dynamics.

(b)

Reconnection can occur where B field lines of opposite direction can meet. The

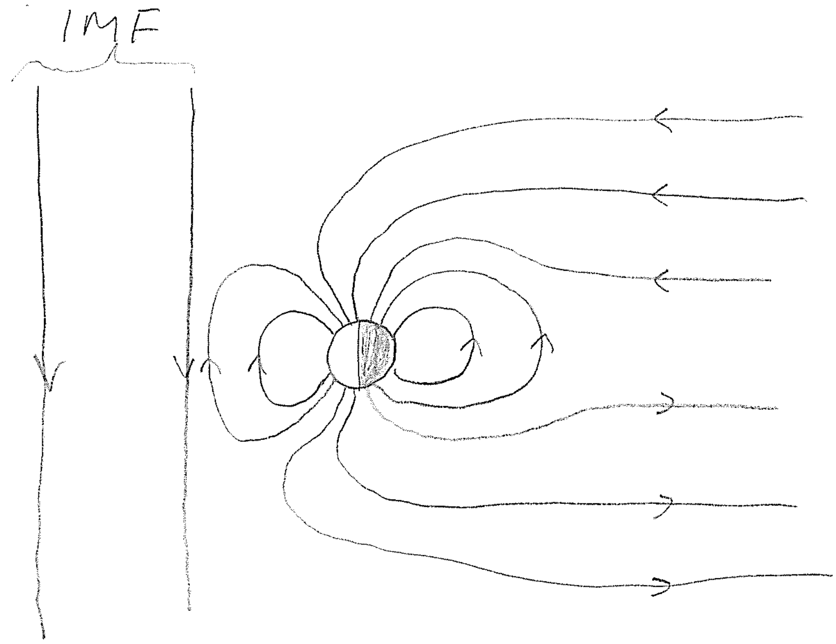


figure at right illustrates that this can happen best when IMF $B_z < 0$. ACE measures the IMF, and we find that this happens for some time intervals, for example:

March 4, 21 - 23 UT

March 5, 04 UT - March 6, 06 UT (most of this time)

This are times at ACE. The delay time from ACE to Earth is calculated from the ACE distance $1.5 \cdot 10^6$ km and the typical solar wind speed 500 km/s as

$$T \sim \frac{1.5 \cdot 10^6}{500} \text{ s} \sim 3000 \text{ s} \sim 50 \text{ min}$$

The delay time is thus ~ one hour, so at the accuracy we have given we indeed need to consider this. Hence, likely reconnection intervals at Earth could be March 4, 22-24 UT, and March 5, 05 UT - March 6, 07 UT.

(c)

The solar wind flow is close to radial, so

$\bar{v} \approx (-400, 0, 0)$ km/s

in GSE coordinates at the time given. The E-field in the frame of the solar wind is close to zero, so for the E-field measured on a s/c we get

$$\bar{E} + \bar{v} \times \bar{B} = 0$$

$$\bar{E} = -\bar{v} \times \bar{B} \approx -\begin{pmatrix} -400 \\ 0 \\ 0 \end{pmatrix} \cdot 10^{3} \times \begin{pmatrix} -3 \\ 2 \\ 2 \end{pmatrix} \cdot 10^{-9} \text{ V/m} =$$

$$= -\begin{pmatrix} 0 \\ 800 \\ -800 \end{pmatrix} \cdot 10^{-6} \text{ V/m} = \begin{pmatrix} 0 \\ -0.8 \\ 0.8 \end{pmatrix} \text{ mV/m}$$

Hence, if ACE had been equipped with an E-field instrument, it would likely have observed an E-field about

$$E_x \approx 0$$

$$E_y = -0.8 \text{ mV/m}$$

$$E_z = 0.8 \text{ mV/m}$$

in GSE coordinates.

T 070309/4)

The s/c temperature will be determined by the radiation balance between absorbed radiation power

$$P_a = A_a \alpha I(r)$$

and emitted power

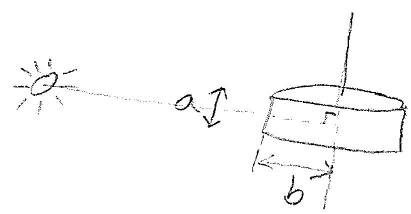
$$P_e = A_e \varepsilon \sigma T^4 .$$

Here $I(r)$ is the solar radiation intensity at distance r from the sun, which is

$$I(r) = I_0 \left(\frac{R}{r}\right)^2$$

where $R = 1$ AU and I_0 is the irradiation at Earth orbit, $I_0 = 1370\ \mathrm{W/m^2}$.

With the given s/c geometry



$$A_a = 2ba = 2 \cdot 1 \cdot 1\ \mathrm{m^2} = 2\ \mathrm{m^2}$$

$$A_e = 2\pi b^2 + 2\pi ab = 2\pi b(a+b) =$$

$$= 2\pi \cdot 1 \cdot 2\ \mathrm{m^2} = 4\pi\ \mathrm{m^2}$$

We thus get

$$\frac{\alpha}{\varepsilon} = \frac{P_a/(A_a I(r))}{P_e/(A_e \sigma T^4)} = \frac{A_e \sigma T^4}{A_a I(r)} = \frac{2\pi \sigma T^4}{I(r)}$$

The s/c must not be cooler than $7°C = 280\ K$ when the radiation is minimal, i.e. at $r = 1.52$ AU, so

$$\frac{\alpha}{\varepsilon} > \frac{2\pi \cdot 5.67 \cdot 10^{-8} \cdot 280^4}{1370/1.52^2} \approx 3.69 .$$

Even when radiation is maximal, at 1 AU, the temperature must not rise above $87°C = 360\,K$,

so

$$\frac{\alpha}{\varepsilon} < \frac{2\pi \cdot 5.67 \cdot 10^{-8} \cdot 360^4}{1370} \approx 4.37.$$

There thus exists an $\frac{\alpha}{\varepsilon}$ range

$$3.69 < \frac{\alpha}{\varepsilon} < 4.37$$

for which the given temperature limits are not exceeded, so the design is possible.

T070309/5)

(a)

Equation of motion:

$$\rho_m \frac{d\bar{v}}{dt} = -\nabla p - \rho_m \bar{g}$$

Assuming stationary equilibrium and $\nabla = \hat{z}\frac{d}{dz}$, where z is the altitude, gives

$$0 = -\frac{dp}{dz}\hat{z} - \rho_m g \hat{z}$$

$$\frac{dp}{dz} = -\rho_m g$$

We put

$$\rho_m = nm.$$

where m is the (mean) molecular mass, and assume an ideal gas, so that

$$p = nKT.$$

Assuming T = constant, we get

$$KT\frac{dn}{dz} = -nmg$$

$$\frac{dn}{n} = -\frac{mg}{KT}dz$$

$$\ln n = -\frac{mgz}{KT} + C$$

$$n = n_0 \exp\left(-\frac{mgz}{KT}\right) \qquad (n_0 = e^C)$$

We can see that the density decays fastest for heavy species.

(b)

Because the primary source of ionizing radiation, i.e. solar EUV radiation, is not there at night. Hence, the plasma density goes down at night, particularly at low heights where recombination is easier thanks to the higher density.

(c)

Because the ionizing radiation does not penetrate far into the atmosphere: it is used up already at high altitudes.