T020814/3)

(a)

$$dI = \sigma n_n(h) I(h)\, dh$$

$$\frac{dI}{I} = \sigma n_n(h)\, dh =$$

$$= \sigma n_0 \exp\left(-\frac{h}{H}\right) dh$$

$$\ln I = -\sigma n_0 H \exp\left(-\frac{h}{H}\right) + C$$

BC: $I = I_0$ for $h \to \infty \Rightarrow C = \ln I_0$

$$I = I_0 \exp\left[-\sigma n_0 H e^{-\frac{h}{H}}\right]$$

$$n_e^2(h) = \frac{a_i}{a_r} n_n(h) I(h)$$

$$n_e(h) = \sqrt{\frac{a_i}{a_r} n_0} \exp\left(-\frac{h}{2H}\right) \exp\left(-\frac{\sigma n_0 H}{2} e^{-\frac{h}{H}}\right) =$$

$$= \sqrt{\frac{a_i n_0}{a_r}} \exp\left(-\frac{h}{2H} - \frac{\sigma n_0 H}{2} e^{-z/H}\right)$$

(b)

$$\frac{dn}{dh} = 0 \Rightarrow 0 = \exp\left(-\frac{h}{2H} - \frac{\sigma n_0 H}{2} e^{-z/H}\right)\left[-\frac{1}{2H} + \frac{\sigma n_0 H}{2H} e^{-z/H}\right]$$

$$\Rightarrow 1 = \sigma n_0 H e^{-z/H}$$

$$\Rightarrow z = -H \ln \frac{1}{\sigma n_0 H} = H \ln(\sigma n_0 H)$$

T020814/4)

(a) μ is adiab inv.

$$\mu = \frac{\frac{1}{2} m v_\perp^2}{B}$$

what is Φ inside?

$$\Phi = I \cdot A$$

$$A = \pi r_g^2 = \pi \left(\frac{v_\perp}{\omega_c}\right)^2 = \frac{\pi v_\perp^2 m^2}{\cancel{e}^2 B^2}$$

$$I = \frac{q}{T_c} = \cancel{\frac{2\pi q}{\omega_c}} = \cancel{\frac{2\pi q \cdot m}{qB}} = \cancel{\frac{2\pi m}{B}}$$

$$\cancel{\Rightarrow \Phi = \pi}$$

$$= \frac{q\omega_c}{2\pi} = \frac{q^2 B}{2\pi m}$$

$$\Rightarrow \Phi = AI = \frac{m v_\perp^2}{2B} = \frac{1}{2}\mu$$

As μ is conserved, so is Φ.

(b) Conserved quantities:

$$E = \frac{1}{2} m v^2$$

$$\mu = \frac{\frac{1}{2} m v^2 \sin^2\alpha}{B}$$

Reaching atm. if $\sin^2 \alpha_a < 1$ in atm.

$$\Rightarrow 1 > \sin^2\alpha_a = \frac{\mu B_a}{\frac{1}{2} m v^2} = \frac{B_a}{B_m} \sin^2\alpha_m$$

T 020814/5/

(b) We use the rocket eqn with $t_{burn} = 0$

$$\Delta v = v_e \ln\left(1 + \frac{m_f}{M_{vehic}}\right)$$

Now let $M = m_p + m_{1f} + m_{1s} + m_{2f} + m_{2s}$

Burning the first stage, we get

$$M_{vehic} = m_p + m_{1s} + m_{2f} + m_{2s}$$

so that

$$\Delta v_1 = v_e \ln\left(1 + \frac{m_{f1}}{m_p + m_{1s} + m_{2f} + m_{2s}}\right)$$

Burning the second stage, we have thrown m_{2s} away so that

$$\Delta v_2 = V_e \ln\left(1 + \frac{m_{f2}}{m_p + m_{2s}}\right)$$

Thus

$$\Delta v_1 + \Delta v_2 = V_e \ln\left[\left(1 + \frac{m_{f1}}{m_p + m_{1s} + m_{2f} + m_{2s}}\right)\left(1 + \frac{m_{2f}}{m_p + m_{2s}}\right)\right]$$

Burning as one stage would give

$$\Delta v = v_e \ln\left(1 + \frac{m_{f1} + m_{f2}}{m_p + m_{2s} + m_{1s}}\right)$$

The difference is

$$\Delta(\Delta v) = V_e \ln \frac{\left(1 + \frac{m_{f1}}{m_p + m_{1s} + m_{2f} + m_{2s}}\right)\left(1 + \frac{m_{2f}}{m_p + m_{2s}}\right)}{1 + \frac{m_{f1} + m_{2f}}{m_p + m_{2s} + m_{1s}}} =$$

$$= v_e \ln \frac{\dfrac{\cancel{m_p + m_{1s} + m_{2f} + m_{2s} + m_{f1}}}{m_p + m_{1s} + m_{2f} + m_{2s}}\left(1 + \dfrac{m_{2f}}{m_p + m_{2s}}\right)}{\dfrac{\cancel{m_p + m_{2s} + m_{1s} + m_{1f} + m_{2f}}}{m_p + m_{2s} + m_{1s}}} =$$

$$= v_e \ln \frac{1 + \dfrac{m_{2f}}{m_p + m_{2s}}}{1 + \dfrac{m_{2f}}{m_p + m_{2s} + m_{1s}}}$$

(c) Scenario 1: one stage

$$m_f = M - m_p - m_s =$$

$$= M - \frac{M}{50} - \frac{M}{10} =$$

$$= M(1 - 0.1 - 0.02) = 0.88\,M$$

$$\Delta v = v_e \ln\left(1 + \frac{0.88}{0.12}\right)$$

~~Bonus:~~

~~$(\Delta v)_{bonus} = v_e \ln 1+$~~

Divide into 2 stages. $m_{f1} = m_{f2} = 0.44\,M$

$m_{s1} = m_{s2} = 0.05\,M$

Bonus:

$$(\Delta v)_{bonus} = v_e \ln \frac{1 + \dfrac{0.44}{0.05 + 0.02}}{1 + \dfrac{0.44}{0.05 + 0.05 + 0.02}} =$$

$\Rightarrow$ 21% increase