Tentamen för kursen Rymdfysik (1FA255)
2015-10-23

Uppsala universitet
Institutionen för fysik och astronomi
Avdelningen för astronomi och rymdfysik
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Answers should be provided in Swedish or English.
Time: 14:00 - 19:00
Allowed tools: Mathematics Handbook (or equivalent), Physics Handbook, enclosed tables and formula sheets, calculator. A bilingual dictionary, for example English-Swedish or English-German, may also be used.
The exam has two parts:

- **Part A** must be satisfactorily solved in order to pass the course. However, if you have passed some of the examlets 1-4, you do not have to solve the corresponding problem in Part A. Which of the examlets you have passed was stated in an e-mail to you in October 20: it is your responsibility to correctly remember this. This part is only graded by pass/fail.
- **Part B** must be solved if you wish to get a higher grade than pass (3 in the Swedish 3-4-5 system, E in ECTS). Grades will depend on the number of points you score on this part.

Part A

1. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can if you wish add comments if you feel the need to do so.
   i. Specific impulse I_{sp} is a measure of the rocket performance, a lower I_{sp} means a better performance.
   ii. Kepler's 2nd law tells us that a body (e.g. a satellite, moon, or planet) in an elliptical orbit around a central body (e.g. moon, planet, or star) moves faster when it is closer to the central body than when it is further away.
   iii. The apoapsis of an orbiting body is the closest distance to the central body.
   iv. A satellite in a geostationary orbit appears motionless, always above the same point on the Earth, to a ground observer.
   v. If we fire our rocket at periapsis then the speed and energy of the satellite increases so it will go into a larger orbit with a larger eccentricity.

   (b) Consider a spacecraft in the shape of a cube, 1 m on each side, with perfect internal heat conductivity. The spacecraft is located at 1 AU from the Sun but far from the Earth. Calculate the equilibrium temperature if $\alpha/\epsilon = 0.4$. The solar constant is 1370 W/m^2 and the Stefan-Boltzmann constant is 5.67×10^{-8} W/m^{2}K^4.

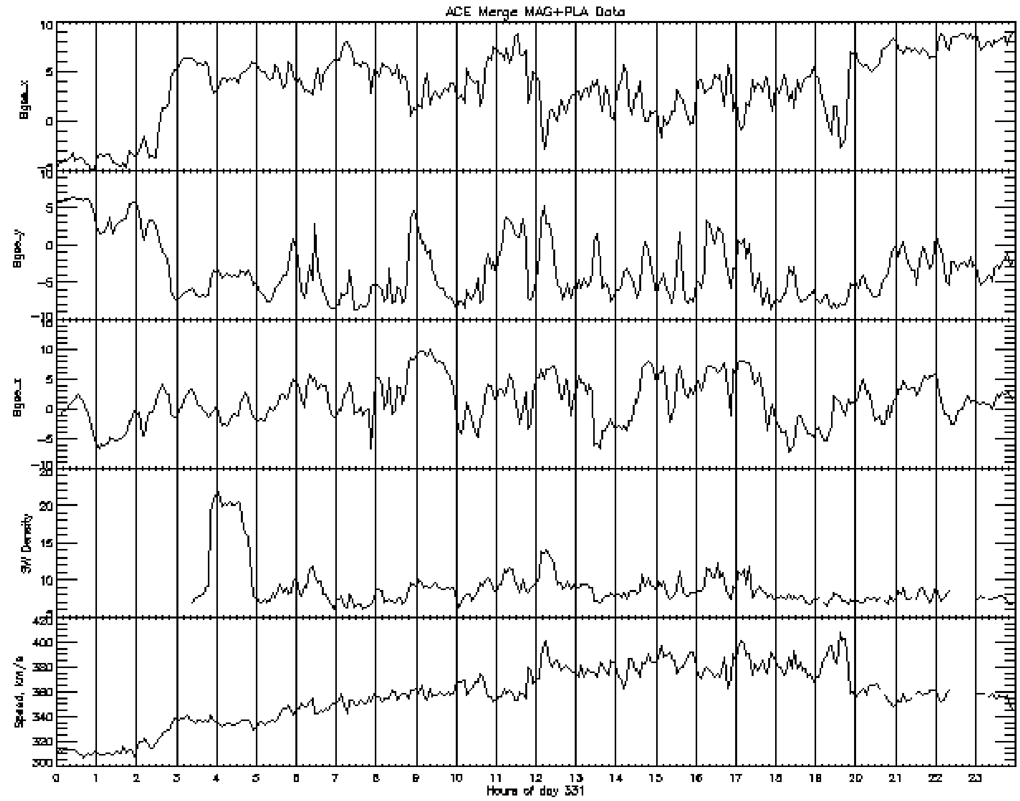

Figure 1: ACE magnetic field and plasma data. The xyz components are given in the GSE system. The plot covers 24 hours (November 27, 2014). From top to bottom, the panels show B_x, B_y, B_z (all in nT), number density (in cm^{-3}), and solar wind speed (in km/s).

2. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so.

   i. Due to quasineutrality, the density of neutral gas is everywhere higher than the plasma density.

   ii. The dominant direction of the interplanetary magnetic field at Earth orbit is along (or opposite to) the direction of the Sun.

   iii. The kinetic energy density in the solar wind typically dominates over the thermal and magnetic energy densities.

   iv. If a magnetic field is frozen into a plasma, two plasma elements which at one time are on the same magnetic field line will always be so.

   v. As the plasma in interplanetary space is tenuous, the interplanetary magnetic field decays with distance r from the Sun at least as fast as $1/r^3$.

   (b) Figure 1 shows a day of magnetic field and solar wind speed measurements from the ACE spacecraft in the solar wind close to the Earth but well outside the bow shock. Estimate the electric field (magntitude and direction) an electric field instrument on ACE would likely have measured at 03:30 (if it existed – there is no electric field instrument on ACE).

3. (a) Are the statements in (i) - (iv) below true or false? You do not need to give any motivation or explanation, but can add comments if you feel the need to do so. For (v), you do need to write some explanation.

   i. If there is an electric field E perpendicular to the magnetic field B in a collisionless plasma, all the plasma (ions and electrons) drift in the same direction, which is perpendicular to E as well as to B.

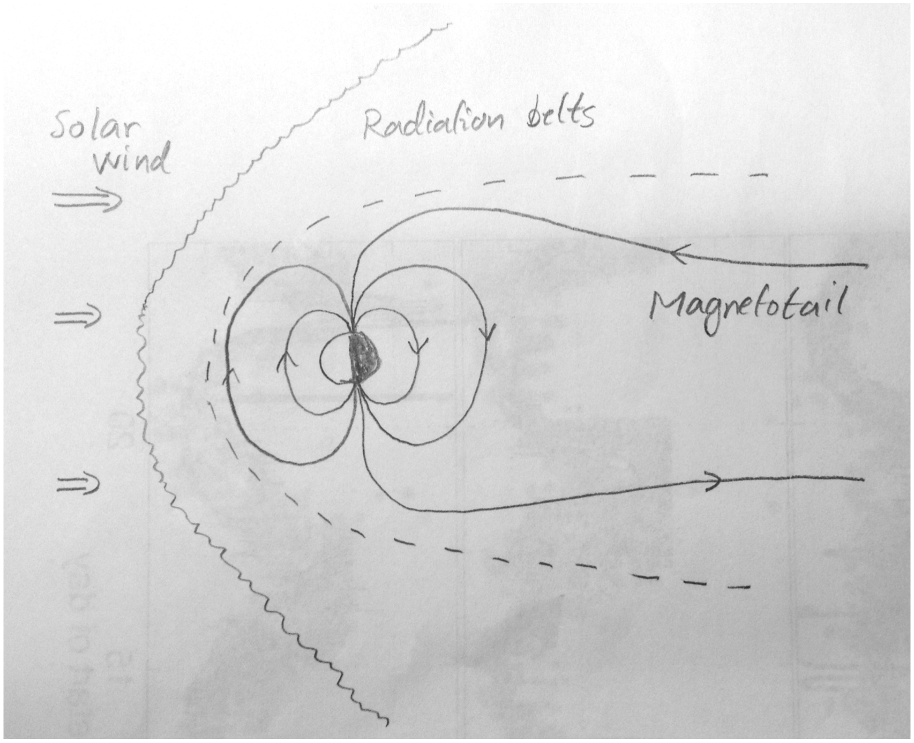

Figure 2: A not very trustworthy sketch of the Earth's magnetosphere.

ii. The mirror point of a charged particle in a dipole field is the point where the pitch angle is $90°$.

iii. The magnetic moment of a charged particle due to its gyromotion is an adiabatic invariant.

iv. Only the magnetic force can do work on a charged particle.

v. Figure 2, identify two qualitative errors and describe (in no more than 10-15 words for each of them) what is the problem. If you think you find more errors you still should write down only two. Quantitative errors, like the magnetosphere being too small, do not count.

(b) An electron has pitch angle $5°$ in the equatorial plane at geocentric distance 3 $\mathrm{R_E}$. Can it reach the ionosphere (which we take to be 100 km above ground)?

4. (a) Are the following statements true or false? You do not need to give any motivation or explanation, but can if you wish add comments if you feel the need to do so.

i. The solar activity varies in an 11-year cycle.

ii. Any planet with an atmosphere, located near a source of ionizing radiation, such as a star, would also have an ionosphere.

iii. In an ionosphere consisting of electrons with a number density n_e and O^+ ions with a number density n_{O+} only, the loss rate is $L = \alpha_r n_e n_{O+}$, where α_r is the recombination rate.

iv. The strongest perpendicular currents of the ionosphere are found in the uppermost part, due to the high plasma density and small collision frequency of this region.

v. Geomagnetic storms are created by changes in the solar wind, often due to solar flares or CMEs.

(b) Why is there a difference between the dayside and nightside plasma density profiles of Earth's ionosphere?

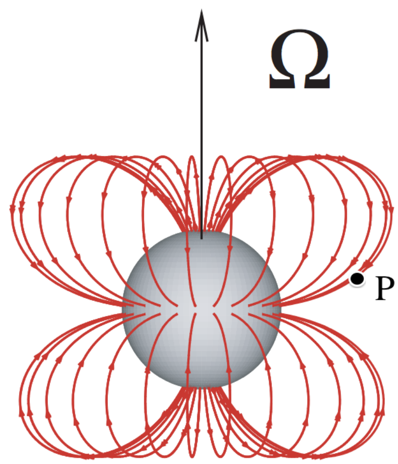

Figure 3: An axisymmetric quadrupole magnetic field. The letter Ω indicates the planetary axis of rotation.

Part B

5. The Kosmos 3M rocket is a two-stage Russian launcher used from 1966 to 2010. The first stage has a specific impulse I_{sp} of 291 s, a dry mass of 1900 kg and a gross mass (dry mass + fuel mass) of 87 200 kg. The second stage has a specific impulse I_{sp} of 293 s, a dry mass of 1440 kg and a gross mass of 20 140 kg. Assume that the burntime is so short that the gravitational term can be neglected and derive Δv for the Kosmos 3M rocket. Compare this with the Δv derived for a single stage rocket with the same exhaust velocity as stage 1 and of the same structure and fuel mass as the whole Kosmos 3M rocket. Which rocket configuration should you chose, single or multi-stage, in order to be able to overcome the escape velocity of Earth if v_{esc} = 11 km/s? (4 p)

6. The yet undiscovered planet Yxplakxrkaxk-6 is in the middle of a magnetic pole reversal, and precisely today has a purely quadrupolar magnetic field, given in spherical polar coordinates by

$$B_r = -\frac{1}{2}B_0\left(\frac{R}{r}\right)^4(2\cos^2\theta - \sin^2\theta) \quad (1)$$

$$B_\theta = -\frac{1}{2}B_0\left(\frac{R}{r}\right)^4 \sin\theta\ \cos\theta \quad (2)$$

$$B_\phi = 0 \quad (3)$$

and illustrated in Figure 3. The radius of the planet is R and the magnetic field strength at the poles is B_0.

(a) Consider an electron with pitch angle $90°$ at the point P in Figure 3. Describe (in words and a sketch or two) its motion in the magnetic field, neglecting effects of gravity and electric fields. (2 p)

(b) Set up and solve the differential equation for the field lines. (3 p)

(c) Is there some latitude where a particle with $90°$ pitch angle can drift along a circle, like in the equatorial plane of the Earth? You do not have to calculate such a latitude, just give convincing arguments for why it does or does not exist. (2 p)

7. Use the solar wind and IMF data in Figure 1.

(a) Discuss how the magnetic field observations compare to what is typically expected. (1 p)

(b) At what time of the day do you think the magnetosphere will be smallest? Clearly stating your assumptions, estimate the stand-off distance, i.e. the distance of the magnetopause from the center of the Earth, at that time. (3 p)

(c) In reality, when they reach the magnetopause, the solar wind and IMF will not have the same properties as seen at ACE. Why? Discuss what this may mean for your estimate of the stand-off distance. (2 p)

8. Assume a neutral atmosphere in hydrostatic equilibrium. From the equation of motion for a neutral gas, derive an expression for how the neutral density is varying with altitude. Assume isothermal conditions (T is constant) and state explicitly all other assumptions you make. (3 p)

Lycka till!

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Galilean transformations:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{B}' = \mathbf{B}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \mathrm{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_\parallel \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \tfrac{ne}{B}\left(\tfrac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \tfrac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \tfrac{ne}{B}\left(\tfrac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \tfrac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\tfrac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \tfrac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_\perp^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu\nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v}_\mathbf{F} = \frac{\mathbf{F}\times\mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan\alpha = v_\perp/v_\parallel$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0}\frac{e^{-r/\lambda_\mathrm{D}}}{r}$$

Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_\mathrm{e}}}$$

Rocket thrust:

$$T = v_\mathrm{e}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_\mathrm{sp} = \frac{\int T\,\mathrm{d}t}{m_\mathrm{fuel}g} = v_\mathrm{e}/g$$

The rocket equation:

$$\Delta v = -gt_\mathrm{burn} + v_\mathrm{e}\ln\left(1 + \frac{m_\mathrm{fuel}}{m_\mathrm{payload+structure}}\right)$$

Emitted thermal radiation power:

$$P_\mathrm{e} = \varepsilon\sigma A_\mathrm{e}T^4$$

Absorbed solar radiation power:

$$P_\mathrm{a} = \alpha A_\mathrm{a}I_\mathrm{rad}$$

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