Tentamen för Rymdfysik I och Rymdfysik MN1
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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well.
Time: 09:00 – 14:00
Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet.

1. Here follows a set of qualitative questions, each of which should be answered in perhaps 5–15 lines of text, possibly an equation or two and maybe a figure.
   (a) What is a plasma? (1 p)
   (b) Why don't satellites just fall down to the ground when the rocket engines are turned off? (1 p)
   (c) What is the plasmasphere? How does it differ from the surrounding regions? From where does its plasma come from? (1p)
   (d) Why is the interplanetary magnetic field stronger than what can be explained by the vacuum dipole fields from the sun and the planets? (1 p)
   (e) What is the solar cycle? How long is it? What happens on the sun during it? What effects does it have on the Earth? (2 p)
   (f) Why is it good to burn off the fuel in a rocket during a launch in as short time as possible? (2 p)
   (g) A geomagnetic substorm may release large amounts of energy in a very short time, part of it going to the aurora. Where, and in what form, has this energy been stored prior to its release? (2 p)

2. When passing the subsolar point, which is the point where the magnetopause is intersected by the Sun-Earth line, a spacecraft records a magnetic field as in Figure 1.
   (a) Estimate the current density (A/m^2) in the magnetopause layer (1 p).
   (b) Estimate the stand-off distance, i.e. the distance of the magnetopause from the center of the Earth. (2 p)
   (c) Estimate the solar wind number density (m^{-3}), if the solar wind speed was 200 km/s. (1 p)
   (d) Estimate the total solar wind dynamic pressure on the Earth's magnetosphere. (1 p)

   One may assume that the geomagnetic field is described by a dipole field all the way out to the magnetopause, that the interplanetary field as well as thermal pressure may be neglected, and that one out of four ions in the solar wind is a He$^+$ while all the rest are protons.

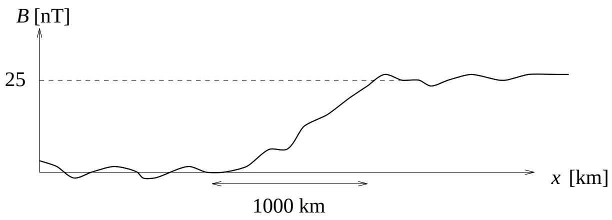

Figure 1: Magnetic field at the magnetopause (problem 2).

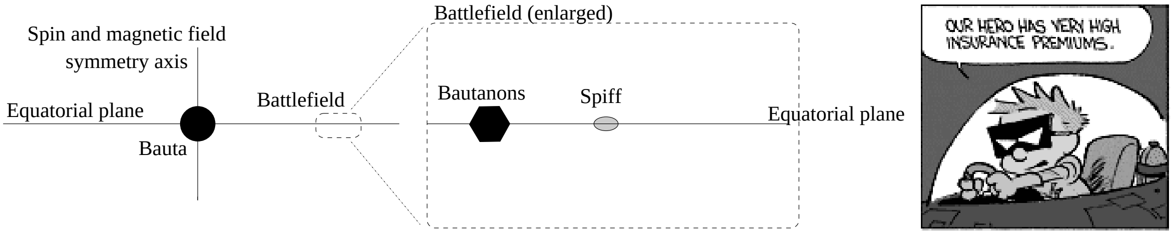

Figure 2: A geometrical (left) and existential (right) analysis of Spiff's encounter with the unpleasant Bautanons (problem 3).

3. The bold spaceman Spiff, famous intergalactic explorer, enters the magnetosphere to the neutron star Bauta. Strangely enough for a neutron star, it turns out to be inhabited by evil creatures called Bautaonons, who are extremly hostile to intruders. They immediately send up a big space battleship with an antimatter gun shoting singly charged negative anti-plutonium ions (242 amu) with an energy of 1 GeV. The neutron star has a very strong, almost dipolar magnetic field (1000 T at the surface at the equator). The two spaceships meet each other in a region near the equator 9 Bauta radii from the surface. Spiff's ship is 100 m radially outward as compared to the big alien warship, and both are located in the equatorial plane (Figure 2). Spiff unfortunately jams his energetic neutral antiparticle beam in this critical moment! The Bautanons aim directly at Spiff and pull the trigger... What happens? Is anybody hit, and in that case, after how long time? The radius of Bauta is only 10 km, since it is a neutron star. Neglect all effects of gravitation. (5 p)

4. The Cassini spacecraft to Saturn (distance from sun 9.54 AU) actually started by going inward in the solar system, passing Venus twice and Earth once before going outward toward Jupiter and Saturn (Figure 3).
   (a) What is the main reason for giving a spacecraft to an outer planet this kind of complicated trajectory? (1 p)
   (b) We have built a spherical Langmuir probe onboard Cassini, (to measure the density and temperature in for example Titan's ionosphere). The probe, which is exposed to the sunlight, is made of titan with a surface coating of titanium nitride, whose absorption and emission coefficients are 0.45 and 0.12, respectively. Neglecting any heat transport to or from the spacecraft body, what are the highest and lowest temperatures we expect the probe to attain during the mission? (3 p)

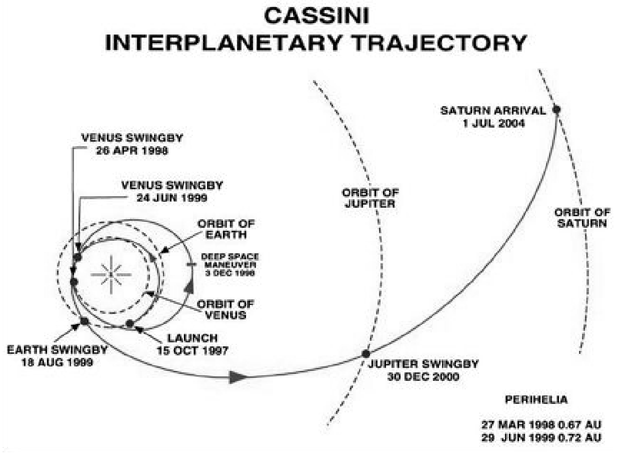

Figure 3: Cassini trajectory to Saturn (problem 4).

5. The diagrams in Figure 4 show the altitude distributions of various dominant constituents as derived from a photo-chemistry model calculation of the Neptunian moon Triton. Both the ion (upper panel) and neutral densities (lower panel) are indicated. The total plasma density, as derived from data from the Voyager-2 Radio Science Subsystem (RSS), is superposed in the upper panel. Note that specifically for N_2, you should multiply the density you get from the figure by 10^7.

   (a) Derive an equation explaining why the concentrations of neutral molecules decrease approximately exponentially with increasing altitude, and why the concentration of heavier constituents (e.g. N_2) decreases slower with altitude than lighter ones (e.g. H_2). State explicitely all assumptions you make. (2 p)

   (b) The Voyager values for the total plasma density are much larger than the values from the model calculations, which are based on the measured intensity of the EUV radiation from the Sun. What could be the physical reason for this anomaly? (1 p)

   (c) Assume the co-rotation velocity of the Neptunian plasma relative Triton's motion to be 40 km/s and that the main ionospheric ion species is N^+. Use the measured maximum ionospheric plasma density from the Voyager data to estimate the induced current density in the ionosphere of Triton as expected from the co-rotation. Estimate the total current flowing through Triton's ionosphere, assuming that the current flows in a 100 km thick altitude layer around the peak ionosphere. (3 p)

   Neptune has a dipole-like magnetic field which is 25 μT on the surface, a radius of 24,750 km, and a rotation period of 15.5 h. Triton, whose radius is 1700 km, can be assumed not have any magnetic field of its own. Assume further that the collision frequencies for the electrons and ions are 200 s^{-1} and 10 s^{-1}, respectively.

Figure 4: Neutral and plasma densities in Triton's ionosphere (see text for problem 5). Molecular nitrogen dominates the atmosphere, so in order to fit this curve together with the other, it has been multiplied by 10^{-7}. The values for N_2 should thus be multiplied by 10^7.

Space Physics Formulas:
Complement to Physics Handbook

Charge density in plasma with charge particle species s:

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\theta}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn\frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v}\times\mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j}\times\mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for ”frozen-in” magnetic field:

$$\mathbf{E} + \mathbf{v}\times\mathbf{B} = 0$$

Ohm’s law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & \sigma_\mathrm{H} & 0 \\ -\sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_{||} \end{pmatrix} \begin{pmatrix} E_\perp \\ 0 \\ E_{||} \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_{||} &= ne^2\left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_\mathrm{c} = \omega_\mathrm{c}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_\perp^2/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu \nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v_F} = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan \alpha = v_\perp / v_\|$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_\mathrm{D}}}{r}$$

Debye length:

$$\lambda_\mathrm{D} = \sqrt{\frac{\epsilon_0 KT}{ne^2}}$$

Plasma frequency:

$$f_\mathrm{p} = \omega_\mathrm{p}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^2}{\epsilon_0 m_\mathrm{e}}}$$

Rocket thrust:

$$T = v_\mathrm{e} \frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_\mathrm{sp} = \frac{\int T \, \mathrm{d}t}{m_\mathrm{fuel} g} = v_\mathrm{e}/g$$

The rocket equation:

$$\Delta v = -g t_\mathrm{burn} + v_\mathrm{e} \ln \left(1 + \frac{m_\mathrm{fuel}}{m_\mathrm{vehicle}}\right)$$

Emitted thermal radiation power:

$$P_\mathrm{e} = \varepsilon \sigma A_\mathrm{e} T^4$$

Absorbed solar radiation power:

$$P_\mathrm{a} = \alpha A_\mathrm{a} I_\mathrm{rad}$$