Solutions to final exam 221018

Space Physics 1FA255

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T221018/5)

(a) Neglecting the thruster firings mean that the blue and the orange dotted segments of the trajectory are Kepler orbits. The Δv provided by the Earth 1 flyby can then be found from comparing the energy of the two orbits. The energy at any point of an elliptic orbit can be written as

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r} = -\frac{GMm}{2a} \qquad (1)$$

where $2a$ is the major axis of the elliptic orbit, which we can measure in Figure 2. However, we find that we cannot measure with such accuracy as to differ between the semimajor axis of the Earth a_0 and of the blue trajectory segment a_1, so we put these to 1 AU. This is also the approximate heliocentric distance of the Earth at the flyby. Denoting the s/c speed just before and after the flyby by v_1 and v_2, respectively, we thus have that $v_1 \approx$ the Earth orbital speed, so by use of the attached table we have

$$v_1 \approx 29.8 \text{ km/s}. \qquad (2)$$

From (1), we then get (with $a_1 \approx a_0 \approx 1$ AU)

$$\frac{1}{2}v_1^2 - \frac{GM}{a_1} = -\frac{GM}{2a_0} \qquad (3)$$

$$\Rightarrow \frac{GM}{a_0} = v_1^2. \qquad (4)$$

In Figure 2, Earth orbit and the blue segment have major axes of about 44 mm while the orange dotted segment has 58 mm. Denoting the semimajor axis of this segment by a_2

we thus have

$$a_2 \approx \frac{58}{44} \cdot 1\,AU = \frac{29}{22}\,AU. \qquad (5)$$

From (1) we find the relations of the energies before and after the flyby:

$$\Delta E = E_2 - E_1 = \frac{1}{2} m (v_2^2 - v_1^2) = -\frac{GmM}{2a_2} + \frac{GmM}{2a_1} \qquad (6)$$

$$\Rightarrow v_2^2 = v_1^2 + \frac{GM}{a_1} - \frac{GM}{a_2} \approx v_1^2 + \frac{GM}{a_0} - \frac{GM}{\frac{29}{22} a_0} =$$

$$= v_1^2 + \frac{GM}{a_0}\left[1 - \frac{22}{29}\right] = v_1^2 + \frac{7}{29}\frac{GM}{a_0} \qquad (7)$$

With (4) & (2), we get

$$v_2^2 \approx v_1^2 \left(1 + \frac{7}{29}\right) = \frac{36}{29} v_1^2 \qquad (8)$$

$$\Rightarrow \Delta v = v_2 - v_1 \approx \sqrt{\frac{36}{29} v_1^2} - v_1 =$$

$$= v_1 \left(\frac{6}{\sqrt{29}} - 1\right) = 29.8 \left(\frac{6}{\sqrt{29}} - 1\right) km/s =$$

$$\approx 3.4\ km/s$$

So, the Δv provided by the first Earth flyby is approximately

$$\boxed{\Delta v \approx 3.4\ km/s}$$

(b) According to Figure 2, the Δv of the relevant orbit maneouvres are 158 m/s and 32 m/s, in total 190 m/s. This is about 6% of the Δv of Earth 1, so the error should of that order, i.e. small.

(c) Without this manoeuvre, Rosetta would just have crossed the orbit of the comet, close to tangentially. Figure 2 shows the semimajor axis of the orbital ellipse before the manoeuvre was larger for the comet than for Rosetta, so the comet had higher orbital energy and thus higher speed at any particular point. Rosetta thus had to accelerate.

T221018/8)

(a) For isotropic outgassing as described in the problem, we have a gas flow velocity

$$\bar{u} = u\hat{r}. \tag{1}$$

Outside the nucleus, there are no sources or losses of gas, so the continuity equation is

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\bar{u}) = 0. \tag{2}$$

We assume steady state $[\partial/\partial t = 0)$, so with (1) we get

$$0 = \nabla\cdot(nu\hat{r}) = \frac{u}{r^2}\frac{\partial}{\partial r}(r^2 n) \tag{3}$$

$$\Rightarrow r^2 n = g(\theta,\varphi) \tag{4}$$

$$\Rightarrow n(r) = \frac{g(\theta,\varphi)}{r^2} \tag{5}$$

where g is an arbitrary function of the angular coordinates θ and φ. For the case of isotropic outgassing g = constant, but we can see that the $1/r^2$ dependence is retained in any given direction also for anisotropic outgassing, as long as the velocity of the gas is strictly radial and there are no interactions between its particles.

(b)

The change of intensity of the EUV radiation in a thin layer $[r, r+dr]$ should be proportional to the intensity $I(r)$ itself, to the number density of the gas $n(r)$ with which the EUV photons can react, and to the thickness dr. Thus

$$dI = a\, n(r)\, I(r)\, dr \tag{5}$$

$$\Rightarrow \frac{dI}{I} = a\, n\, dr \tag{6}$$

With (4) we get

$$\frac{dI}{I} = a \frac{g}{r^2} dr. \tag{7}$$

With intensity I_0 far away $(r \to \infty)$, we have

$$\int_{I(r)}^{I_0} \frac{dI}{I} = ag \int_r^{\infty} \frac{dr}{r^2} \tag{8}$$

$$\left[\ln I\right]_{I(r)}^{I_0} = ag \left[-\frac{1}{r}\right]_r^{\infty} \tag{9}$$

$$\ln \frac{I(r)}{I_0} = -\frac{ag}{r} \tag{10}$$

$$\Rightarrow \boxed{I(r) = I_0 \exp\left(-\frac{ag}{r}\right)} \tag{11}$$

where a and g are constants.