

T050311/1)

1:1 A, B, C

1:2 A, B

1:3 A

1:4 B (A is not true - the rad belt particles do not corotate with the Earth)

1:5 A (B is not true - the interplanetary B-field forms a spiral-like structure, but the solar wind flow is essentially radial)

1:6 A (any power line problems related to violent events on the sun are due to changing geomagnetic currents, which have more to do with solar wind disturbances, for which the typical timescale to reach the Earth is days, not 8 minutes, so B is not fully correct)

1:7 B, C

1:8 A, C

1:9 A, C

1:10 B

T050311/2)

As the monsters shoot the ions at an angle  $\theta_Q$  to the magnetic field, this will be the pitch angle of the ions at point Q:

$$\alpha_Q = \theta_Q \quad (1)$$

From the conservation of energy,

$$E = \frac{1}{2}mv^2 = \text{constant}, \quad (2)$$

and the adiabatic invariance of the orbital magnetic moment,

$$\mu = \frac{\frac{1}{2}mv_{\perp}^2}{B} = \frac{\frac{1}{2}mv^2 \sin^2 \alpha}{B} = \frac{E \sin^2 \alpha}{B} = \text{const.} \quad (3)$$

it follows that  $\frac{\sin^2 \alpha}{B}$  also is constant, so that

$$\frac{\sin^2 \alpha_Q}{B_Q} = \frac{\sin^2 \alpha_P}{B_P} = \frac{\sin^2 \alpha_J}{B_J} \quad (4)$$

The ions are magnetically mirrored at the point where  $\sin^2 \alpha = 1$ . If this point is between Q and P, the ions will never reach P but turn back to Q (case c). If the mirror point lies between I and P, the ions will reach P, mirror and go back to Q (case B). Finally, if the mirror point is below J, the ions are lost in the ionosphere, so that they reach P without ever coming back to Q (case a). We have in this case

$$\frac{1}{B_m} = \frac{\sin^2 \alpha_Q}{B_Q} = \frac{\sin^2 \theta_Q}{B_Q}$$

$$\Rightarrow B_m = \frac{B_Q}{\sin^2 \theta_Q} = \frac{B_Q}{\sin^2 30^\circ} = 4B_Q = 4 \mu\text{T}$$

Hence,  $B_m < B_P$ , so the ions mirror at a point between P and J (case c). Swift survives while the

T050311/3)

(a) The density is very low and the speed is high compared to typical values of  $5 \text{ cm}^{-3}$  and  $400 \text{ km/s}$ , but the solar wind is very variable and the values are still quite normal.

(b) We assume a pressure balance between the solar wind dynamic pressure (momentum flux),

$$P_{\text{sw}} \sim n m v^2 \quad (1)$$

and the magnetic pressure inside the magnetosphere,

$$P_B = \frac{B^2}{2\mu_0} \quad (2)$$

(neglecting thermal pressure in the magnetosphere).

Assuming a dipole field,

$$B = B_0 \left( \frac{R_E}{r} \right)^3 \quad (3)$$

we get

$$n m v^2 = \frac{B_0^2}{2\mu_0} \left( \frac{R_E}{r} \right)^6 \quad (4)$$

$$r = R_E \left[ \frac{B_0^2}{2\mu_0 n m v^2} \right]^{1/6} \quad (5)$$

Here

$$n = 0.4 \text{ cm}^{-3}$$

$$v = 607 \text{ km/s}$$

are given. For an estimate, we may take the typical solar wind particle mass to be

$$m = m_{\text{proton}},$$

and on the ground we have

$$B_0 = 30 \mu\text{T}.$$

Hence, we get

$$r \approx \left[ \frac{(30 \cdot 10^{-6})^2}{2 \cdot 4\pi \cdot 10^{-7} \cdot 0.4 \cdot 10^6 \cdot 1.67 \cdot 10^{-27} \cdot (604 \cdot 10^3)^2} \right] R_E$$
$$\approx 11 R_E$$

(c) The out-of-ecliptic ( $z$ ) component of the IMF controls reconnection at the magnetopause. For southward IMF ( $B_z < 0$ ), the IMF and the geomagnetic field are oppositely directed, and reconnection can occur at the dayside magnetopause. This causes transport of magnetic flux to the tail, where magnetic energy can be piled up until released in a substorm. Hence, substorms are more common during IMF  $B_z < 0$ , which is the reason for why this component is interesting.

T050311/4)

(a) Equation of motion for the neutral gas:

$$\rho_m \frac{d\bar{v}}{dt} = -\nabla p + \rho_m \bar{g} \quad (1)$$

Assume a static atmosphere ( $\bar{v} = 0$ ,  $\frac{d}{dt} = 0$ ) which is horizontally stratified ( $\nabla = \hat{z} \frac{d}{dz}$ ), an ideal gas ( $p = nKT$ ), isothermal conditions ( $\nabla T = 0$ ), constant gravitational field  $\bar{g} = -g\hat{z}$  and put  $\rho_m = mn$ , where  $m$  is the mean molecular mass in the gas. Equation (1) then becomes

$$0 = -\hat{z} \frac{d}{dz} (nKT) - mng\hat{z} \quad (2)$$

$$KT \frac{dn}{dz} = -mgn \quad (3)$$

$$\frac{dn}{n} = -\frac{mg}{KT} dz \quad (4)$$

Integrate:

$$\ln \frac{n}{n_0} = -\frac{mgz}{KT} \quad (5)$$

$$n(z) = n_0 \exp\left(-\frac{mgz}{KT}\right) \quad (6)$$

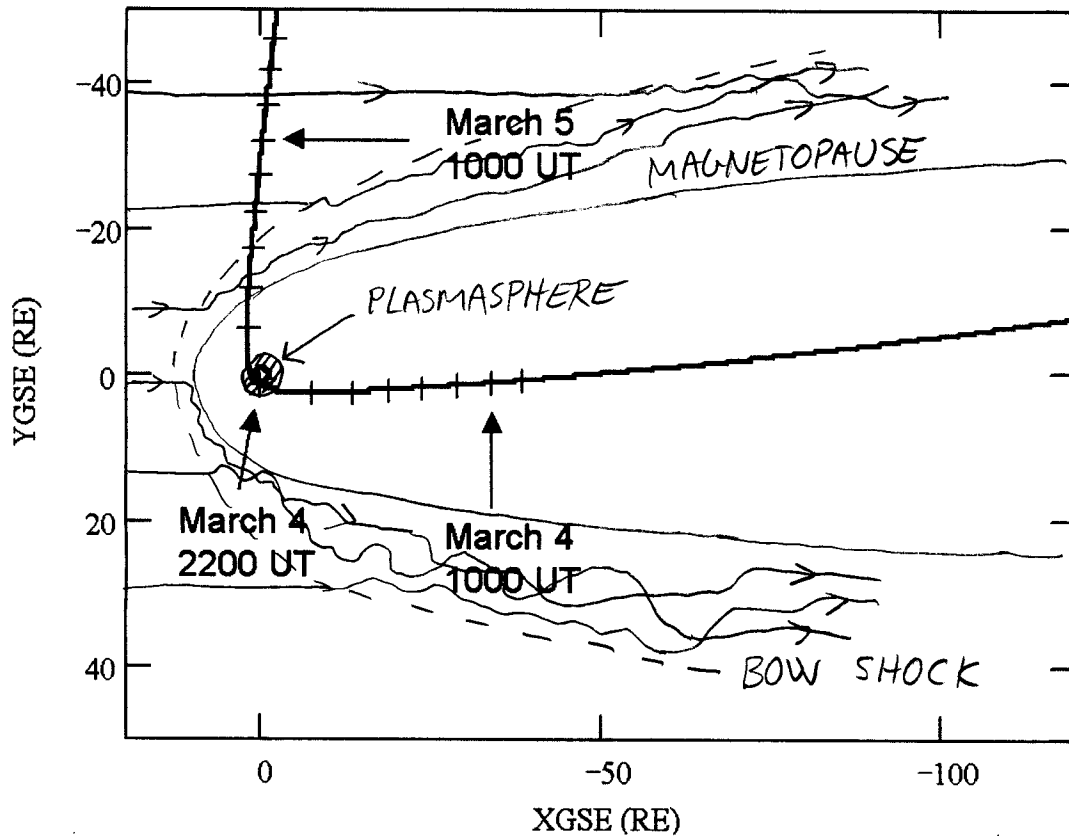
där  $n_0 = n(0)$ . It follows that heavier species have a steeper decline in concentration with altitude  $z$ , and as  $m_O < m_{N_2} < m_{O_2}$ , we have the desired result.

(b) Ionization by solar radiation occurs only in daytime, while recombination goes on day and night. The electron density therefore drops at night on these altitudes.

T 050311/5)

(a)

→ solar wind stream lines



(b) Energy conservation:

$$\frac{1}{2}mv^2 - \frac{GmM}{r} = \text{constant} = E \quad (1)$$

where  $m$  is spacecraft mass,  $M = \text{Earth mass}$ ,  
 $r = \text{geocentric distance}$ ,  $v = \text{spacecraft speed}$ .

Far out ( $r \rightarrow \infty$ ):

$$E = \frac{1}{2}mv_0^2, \quad v_0 = 3.9 \text{ km/s} \quad (2)$$

At  $r = 1900 \text{ km}$

$$\frac{1}{2}mv^2 = E + \frac{GmM}{r} = \frac{1}{2}mv_0^2 + \frac{GmM}{r} \quad (3)$$

$$v = \sqrt{v_0^2 + \frac{2GM}{r}} = \quad (4)$$

$$= \sqrt{(3.9 \cdot 10^3)^2 + \frac{2 \cdot 6.67 \cdot 10^{-11} \cdot 6 \cdot 10^{24}}{(1900 + 6371.2) \cdot 10^3}} \text{ m/s}$$

$$\approx 10,6 \text{ km/s}$$

(c) We use the rocket equation, neglecting the  $gt_{\text{burn}}$  term:

$$\Delta v = v_e \ln \left( 1 + \frac{m_f}{m_v} \right) \quad (5)$$

For the real Rosetta launch,

$$m_f = \text{fuel mass} = 637 \text{ t}$$

$$m_v = \text{vehicle mass} = (749 - 637) \text{ t} = 112 \text{ t}$$

$$\Delta v = 3.5 \text{ km/s}$$

and we may calculate

$$v_e = \frac{\Delta v}{\ln \left( 1 + \frac{m_f}{m_v} \right)} \quad (6)$$

To get  $\Delta v' = 3.9 \text{ km/s}$  with the same  $m_v$  and  $v_e$ , we get

$$\Delta v' = v_e \ln \left( 1 + \frac{m_f'}{m_v} \right)$$

$$1 + \frac{m_f'}{m_v} = \exp \left( \frac{\Delta v'}{v_e} \right) = \exp \left( \frac{\Delta v'}{\Delta v} \ln \left[ 1 + \frac{m_f}{m_v} \right] \right)$$

$$= \left[ 1 + \frac{m_f}{m_v} \right]^{\frac{\Delta v'}{\Delta v}} \quad (7)$$

$$m_f' = m_v \left( \left[ 1 + \frac{m_f}{m_v} \right]^{\frac{\Delta v'}{\Delta v}} - 1 \right) = \quad (8)$$

$$= 112 \left( \left[ 1 + \frac{637}{112} \right]^{\frac{3.9}{3.5}} - 1 \right) t \approx 819 t$$

A rough estimate that is that we would have needed

$$m_f' - m_f = (819 - 637) t \approx 182 \text{ tons}$$

more fuel to get a speed of 3.9 km/s.

(d) Only radiation is important under the assumptions given in the problem.

Emitted radiation power:

$$P_e = \epsilon \sigma A_e T^4$$

$$A_e = \text{total area} = 4\pi r^2$$

Absorbed power:

$$P_a = \alpha I A_a$$

$$A_a = \text{area projected to the sun} = \pi r^2$$

$$I = 1367 \text{ W/m}^2 \text{ (solar intensity at 1 AU)}$$

$$\text{Energy balance} \Rightarrow P_e = P_a \Rightarrow$$

$$T^4 = \frac{\alpha I A_a}{\epsilon \sigma A_e} = \frac{\alpha I}{4 \epsilon \sigma}$$

$$T = \left[ \frac{0.47 \cdot 1367}{4 \cdot 0.1 \cdot 5.67 \cdot 10^{-8}} \right]^{1/4} \text{ K} \approx 410 \text{ K} \approx 137^\circ \text{C}$$