Tentamen för Rymdfysik I och Rymdfysik MN1
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Uppsala universitet
Institutionen för astronomi och rymdfysik
Anders Eriksson

Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well. Answers may of course be given in Swedish or English, according to your own preference.

Time: 9:00 – 14:00

Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet.

Solutions

1. Here follows a set of qualitative questions, each of which should be answered in perhaps 5–15 lines of text, possibly an equation or two and maybe a figure.

   (a) What makes plasma physics so much more complicated than the physics of a neutral gas? (1 p)

   (b) What is the meaning of the concept ”frozen-in magnetic field”, and in what circumstances is it appliccable? (1 p)

   (c) Name at least two planets in the solar system, except the Earth, that should have magnetospheres. (1 p)

   (d) You have probably heard that ”in space there are no sounds, as there is nothing for sound waves to propagate in”. Well, now when you know that ”empty space” is not empty, what do you think? Are there sounds in space? Intelligent discussion is more important here than exactly right or wrong. (1 p)

   (e) What is the solar corona? What is (in science in general and in this course in particular) regarded as the most important unsolved problem regarding the corona? (2 p)

   (f) An argument sometimes heard goes as this: ”It is impossible to gain energy without using any fuel by so-called gravity assist manouvres, in which a spacecraft flies by close to a planet. As gravity is a conservative force, the speed of the spacecraft is the same when going away from the planet as when coming in towards it. Thus gravity assist is impossible." Still gravity assist obviously works, without any active propulsion from rocket engines or solar sails or anything, as shown by many interplanetary spacecraft. How? (2 p)

(g) Draw a **large** (use a separate paper) and **clear** sketch of the Earth's magnetosphere, indicating:

   i. Representative geomagnetic field lines, with direction
   ii. Representative solar wind flow lines, with direction
   iii. The bow shock
   iv. The magnetopause
   v. The plasmasphere

   (2 p)

Solution:

(a) The particles constituting the plasma, or at least a significant fraction of them, are electrically charged, and they may thus interact through electric and magnetic fields. In a true plasma, the individual particle-particle interactions are unimportant, but the collective effects of the motion of many particles creates electromagnetic fields which in their turn govern the motion of the particles. This is what makes plasma physics complicated – and fun!

(b) "Frozen-in" refers to the fact that the magnetic flux is tied to the plasma motion, so that it is possible to treat the magnetic field lines as physical objects, much like strings of well-boiled spaghetti in jelly. The concept can only be applied for long time scales (much above the gyroperiod) and long spatial scales (much longer than the gyroradius).

(c) Clear cases: the giants (Jupiter, Saturn, Uranus, Neptune). Less clear case: Mercury (it is so small that it at times probably does not go outside the planet surface on the dayside). Even less clear case: Mars (having very weak and inhomogeneous, "patchy", magnetic field of its own).

(d) The space plasma has a pressure, and thus support sound-like waves. The properties of these waves are complicated by the electric interactions between the plasma particles and thus changed from ordinary sound waves, but they still have pressure variations and should thus in principle be audible to a sufficiently sensitive ear.

(e) The solar corona is the uppermost layer of the solar atmosphere (if you do not consider the solar wind as a part of that atmosphere), usually considered to stretch out tens of solar radii from the sun. The most remarkable feature of the corona is its high temperature: millions of K, while the underlying chromosphere and photosphere have temperatures several order of magnitudes lower. The mystery sought in the problem text is the heating of the corona to these high temperatures, which is still unknown (at least in its details).

(f) In the frame of reference of the planet, the energy of the spacecraft is indeed conserved. However, as the planet itself is moving around the sun, a spacecraft can indeed gain (or loose) energy when passing the planet. Total energy is of course preserved, the planet changing its motion imperceptibly in response to the weak gravitational pull of the spacecraft.

(g)

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2. It would be very interesting to send a spacecraft to do measurements of the plasma and the electromagnetic fields in the solar corona, for instance for solving the problem I hope you have mentioned in the solution to Problem 1e. However, going so close to the sun poses significant technical problems, above all thermal problems. One of the ideas of how to keep cool is to build a conical spacecraft with the top of the cone toward the sun. Assuming no internal dissipation of energy, derive an expression for the equilibrium temperature as a function of cone top (half-)angle and distance to the sun for such a spacecraft. What top angles would be needed at a distance of 40 solar radii from the sun in order to get down to temperatures of 400°C and 50°C, respectively, if the surface properties are $\alpha = 0.56$ and $\epsilon = 0.37$ (corresponding to e.g. unpolished steel)? The total solar luminosity is $3.9 \cdot 10^{26}$ W, and the solar radius is $696\,000$ km. (4 p)

Solution:

In equilibrium, the sunlight radiation power absorbed by the spacecraft,

$$P_{\mathrm{a}} = A_{\mathrm{a}} \alpha I_{\mathrm{rad}}, \tag{1}$$

must be equal to the emitted thermal radiation power,

$$P_{\mathrm{e}} = A_{\mathrm{e}} \epsilon \sigma T^4, \tag{2}$$

so that

$$T = \left(\frac{\alpha}{\epsilon} \frac{A_{\mathrm{a}}}{A_{\mathrm{e}}} \frac{I_{\mathrm{rad}}}{\sigma} \right)^{1/4}. \tag{3}$$

The first fraction in the parenthesis is given by the constants provided in the problem. To find the area ratio $A_{\mathrm{a}}/A_{\mathrm{e}}$, we first note that from the sun, the spacecraft looks like a circle of some radius r: thus the absorbtion area is

$$A_{\mathrm{a}} = \pi r^2. \tag{4}$$

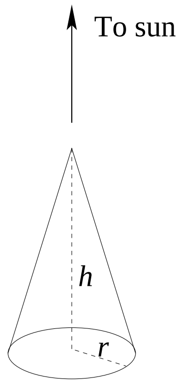

Figure 1: Geometry of the conical spacecraft.

The emission area A_e is the full area of the spacecraft, i.e. of the cone including its bottom. With a circular radius r and a top half-angle ϕ (Figure 1), the height of the cone is

$$h = r/\tan\phi, \tag{5}$$

so that its mantle area is

$$\begin{aligned} A_\mathrm{m} &= \pi r\sqrt{r^2+h^2} = \pi r^2\sqrt{1+\frac{1}{\tan^2\phi}} = \pi r^2\sqrt{1+\frac{\cos^2\phi}{\sin^2\phi}} = \\ &= \pi r^2\sqrt{\frac{\sin^2\phi}{\sin^2\phi}+\frac{\cos^2\phi}{\sin^2\phi}} = \pi r^2/\sin\phi. \end{aligned} \tag{6}$$

The total emissive area we get by adding this to the bottom area, so that

$$A_\mathrm{e} = \pi r^2\left(1+\frac{1}{\sin\phi}\right). \tag{7}$$

Hence the area ratio in (3) is

$$\frac{A_\mathrm{a}}{A_\mathrm{e}} = \frac{1}{1+\frac{1}{\sin\phi}}. \tag{8}$$

To get the last unknown parameter in equation (3), the solar radiation intensity at distance R from the sun, we divide the total power of the sun by the area of a sphere of radius R, i.e.

$$I_\mathrm{rad} = \frac{P_\mathrm{rad}}{4\pi R^2}. \tag{9}$$

Putting (8) and (9) into (3), we get

$$\boxed{T = \left(\frac{\alpha}{\epsilon} \frac{1}{1+\frac{1}{\sin\phi}} \frac{P_{\mathrm{rad}}}{4\pi\sigma R^2} \right)^{1/4}} \tag{10}$$

To find the angles needed to keep the spacecraft at the stated temperatures, we solve equation (10) for $\sin\phi$, getting

$$\sin\phi = \frac{1}{\frac{\alpha P_{\mathrm{rad}}}{4\pi\sigma R^2 T^4} - 1}. \tag{11}$$

Stefan-Boltzmann's constant σ is a table value, and all other numerical values are provided in the text. For $T = 400°\mathrm{C} = 673$ K, we get a top half-angle of $14°$, which may be possible to build, but this temperature is too high for normal electronics. On the other hand, the more comfortable temperature of $T = 50°\mathrm{C} = 323$ K gives $0.6°$, which is quite an unreasonably narrow geometry for a spacecraft. One may consider putting a conical parasoll on some booms protruding from the spacecraft, so that the parasoll can be several hundred $°\mathrm{C}$ without hurting the spacecraft.

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3. (a) Derive an expression for the electron number density n_{e} in an ionosphere as a function of altitude h above the ground assuming that:
   - the neutral gas has the same temperature and composition on all heights while its number density varies as $n_{\mathrm{n}}(h) = n_0 \exp(-h/H)$, where $H = KT/(mg)$, m is the mean molecular mass and n_0 is the atmospheric number density at the ground,
   - the intensity I of the ionizing radiation increases with altitude as determined by $\mathrm{d}I = \sigma n_{\mathrm{n}}(h) I(h)\, \mathrm{d}h$,
   - ionization and recombination balances each other, so that $a_{\mathrm{i}} n_{\mathrm{n}}(h) I(h) = a_{\mathrm{r}} n_{\mathrm{e}}^2(h)$.

   Here, σ, a_{i} and a_{r} are constants, K is Boltzmann's constant, and g is the acceleration of gravity (assumed constant with altitude). (4 p)

   (b) Derive an expression for the altitude of the maximum electron density in (a) only depending on the constants H, σ and n_0. (2 p)

Solution:

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4. Consider an auroral electron (energy in the 10 keV range) at some point above the auroral zone.

(a) Show, for instance by using an adiabatic invariant, that the particle is moving on the surface of a magnetic flux tube (which means that you shall show that the total magnetic flux inside the particle gyroorbit is constant). (2 p)

(b) Derive the condition on the magnetic field strengths (locally and down in the atmosphere) that must be satisfied for the particle to reach the atmosphere before it is mirrored. (2 p)

Solution:

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5. The total mass launched by a rocket can be written

$$M = m_\mathrm{p} + m_\mathrm{f} + m_\mathrm{s}$$

where m_p is the payload that we actually want to put into orbit, m_f is the fuel and m_s is the structural mass, i.e. the mass of the rocket itself.

(a) Why is it at all good to divide a rocket into several stages? Describe in words. (1 p)

(b) Show that dividing the rocket into two stages gives an additional Δv

$$(\Delta v)_\mathrm{bonus} = v_\mathrm{e} \ln \frac{1 + \frac{m_{2\mathrm{f}}}{m_{2\mathrm{s}} + m_\mathrm{p}}}{1 + \frac{m_{2\mathrm{f}}}{m_{1\mathrm{s}} + m_{2\mathrm{s}} + m_\mathrm{p}}}$$

as compared to using the same fuel and structure mass in one single stage. (3 p)

(c) Consider a one-stage rocket launch with $m_\mathrm{p} = 0.02\,M$ and $m_\mathrm{s} = 0.1\,M$. For the same payload, how much does Δv increase by dividing the rocket into two identical stages? Assume that the ratio of fuel to structure mass is the same for each stage and also for the one-stage rocket you compare to. The answer should be given in per cent of the one-stage Δv. (2 p)

We assume that the rockets burn very quickly, so that the term $g t_\mathrm{burn}$ in the rocket equation can be neglected.

Solution: