Tentamen för Rymdfysik I och Rymdfysik MN1
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Please write your **name** on **all** papers, and on the first page your **address, e-mail** and **phone number** as well.

Time: 14:00 – 19:00

Allowed tools: Mathematics Handbook, Physics Handbook, enclosed formula sheet.

Solutions

1. Here follows a set of qualitative questions, each of which should be answered in perhaps 5–15 lines of text, possibly an equation or two and maybe a figure.

   (a) What is gyromotion? (1 p)

   (b) What is the solar wind? (1 p)

   (c) The solar wind plasma has a temperature of typically 10 eV, i.e. around 100 000°C – so why doesn't an interplanetary spacecraft immediately melt or burn to ashes? (1 p)

   (d) What is the meaning of the concept ”frozen-in magnetic field”, and in what circumstances is it appliccable? (1 p)

   (e) Draw a graph of the electron number density n_e in the Earth's ionosphere as a function of altitude h from ground level up to the topside ionosphere. Explain why $n_e(h)$ looks like you have drawn. (The absolute numerical values of n_e are not so important, but you should have reasonable values on the h axis.) (2 p)

   (f) Which of the following bodies can be expected to have an ionosphere:

      i. Venus
      ii. the Moon
      iii. the Milky Way galaxy

iv. the Wind satellite (orbiting the Earth well outside Earth's magnetosphere)

Motivate your answer in each case. It is the way you think and reason that is important – a well motivated but wrong answer may be accepted, while a short "yes" or "no" answer certainly will not. (2 p)

(g) Draw a **large** (use a separate paper) and **clear** sketch of the Earth's magnetosphere, indicating:

i. Representative geomagnetic field lines, with direction
ii. Representative solar wind flow lines, with direction
iii. The bow shock
iv. The magnetopause
v. The Van Allen radiation belts

(2 p)

Solution:

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2. The Cluster spacecraft have recently detected electrons with energies between 100 eV and 1 keV just inside the frontlobe magnetopause of Earth at a distance of about 12 Earth radii from Earth's center. The electrons were observed when the spacecraft were about 30° above the (magnetic) equatorial plane ($\theta = 60°$).

(a) Adopt a dipole field approximation and calculate the magnetic field strength at the Cluster location. The magnetic field strength at the surface of Earth near the equator is about 30 μT. (1 p)

(b) The electrons are found only for pitch angles within $\pm 10°$ around the geomagnetic field direction and are counterstreaming (i.e. streaming in both directions with respect to the geomagnetic field direction). At what magnetic field strengths will these electrons mirror? At what altitudes above the Earth surface does the mirroring take place? The radius of the Earth is 6370 km. (3 p)

Solution:

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3. Consider the following model of the magnetic field in the central part of the geomagnetic tail:

$$\mathbf{B}(\mathbf{r}) = \begin{cases} -B_0 \hat{\mathbf{x}} & , \quad z < -a \\ B_0\, \hat{\mathbf{x}}\, \frac{3\,a^2\,z - z^3}{2\,a^3} & , \quad -a \leq z \leq a \\ B_0 \hat{\mathbf{x}} & , \quad z > a \end{cases}$$

where $B_0 = 1$ nT, $a = 2000$ km and the coordinates are defined as in Figure 1.

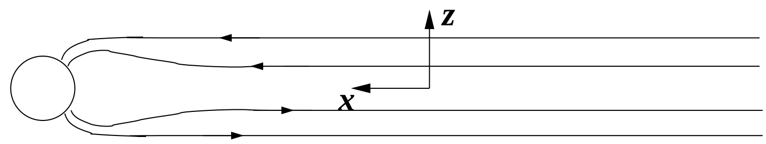

Figure 1: Idealized geometry of the relevant part of the geomagnetic tail.

(a) Calculate the current density $\mathbf{j}(\mathbf{r})$ and the magnetic force density $\mathbf{j}(\mathbf{r}) \times \mathbf{B}(\mathbf{r})$ (magnitudes and directions as functions of position). Also calculate their numerical values at $z = 0$. (3 p)

(b) Now consider what happens if an instability appears in the region $-a < x < a$, $-10\,a < y < 10\,a$, $-a < z < a$ so that the resistivity in this region includes drastically. Instead of flowing through this region as before, the current now instead closes through field aligned currents and the ionosphere. How strong (in units of ampères) will these field-aligned currents be? Is this example relevant for any phenomenon in Earth's magnetosphere? (3 p)

Solution:

(a) Ampère's law gives

$$\mathbf{j} = \frac{1}{\mu_0} \nabla \times \mathbf{B}. \tag{1}$$

In the constant field in the regions $|z| > a$ this obviously is zero, while for the region $|z| < a$ we get

$$\mathbf{j} = \frac{1}{\mu_0} \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_0 \frac{3a^2 z - z^3}{2a^3} & 0 & 0 \end{vmatrix} = \frac{3B_0}{2\mu_0 a^3}(a^2 - z^2)\hat{\mathbf{y}} \tag{2}$$

and

$$\mathbf{j} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ 0 & \frac{3B_0(a^2 - z^2)}{2\mu_0 a^3} & 0 \\ B_0 \frac{3a^2 z - z^3}{2a^3} & 0 & 0 \end{vmatrix} = -\frac{3B_0^2}{4\mu_0 a^6}(a^2 - z^2) z (3a^2 - z^2)\hat{\mathbf{z}}. \tag{3}$$

Numerically, we get

$$\mathbf{j}(x, y, 0) = \frac{3B_0}{2\mu_0 a}\hat{\mathbf{y}} = \frac{3 \cdot 1 \cdot 10^{-9}}{2 \cdot 4\pi \cdot 10^{-7} \cdot 2000 \cdot 10^3}\, \hat{\mathbf{y}}\mathrm{A/m^2} \approx 0.6\, \hat{\mathbf{y}}\; \mathrm{nA/m^2}, \tag{4}$$

while the factor z appearing in the right hand side of (3) obviously ensures that

$$\mathbf{j}(x, y, 0) \times \mathbf{B}(x, y, 0) = 0. \tag{5}$$

(b) The field aligned currents will carry all the current that initially flowed through the region considered. As the current flow was in the y direction, the total current is given by

$$\begin{aligned} I &= \int_{-a}^{a}\int_{-a}^{a} j(x,y,z)\,\mathrm{d}x\,\mathrm{d}z = 2a\int_{-a}^{a}\frac{3B_0}{2\mu_0 a^3}(a^2-z^2)\,\mathrm{d}z = \\ &= \frac{3B_0}{\mu_0 a^2}\left[a^2 z - z^3/3\right]_{-a}^{a} = \frac{3B_0}{\mu_0 a^2}\cdot\frac{4a^3}{3} = \frac{4aB_0}{\mu_0} \end{aligned} \quad (6)$$

(There is a simpler way to get this – the surface current, in units of A/m2, between two regions of different but parallel magnetic fields is $K = \Delta B/\mu_0$, so in this case where the magnetic field changes from $B_0\hat{\mathbf{x}}$ to $-B_0\hat{\mathbf{x}}$ we have $K = 2B_0/mu_0$; multiplying by the interval in x, that is $2a$, results in $I = 2aK = 4aB_0/\mu_0$ as above.)
Numerically, we get

$$I = \frac{4\cdot 2000\cdot 10^3\cdot 1\cdot 10^{-9}}{4\pi\cdot 10^{-7}}\mathrm{A} \approx 6\ \mathrm{MA}. \quad (7)$$

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4. Each of the four Cluster spacecraft, orbiting the Earth with apogee close to 20 Earth radii, is approximately a cylinder of radius 1.5 m and height 1 m, with the symmetry axis perpendicular to the direction of the sun. The mantle areas are covered with solar panels, while the top and bottom sides are mainly covered with a thermal blanket. Estimate the equilibrium temperature (in °C) of the satellites, assuming perfect thermal conductivity within the spacecraft and using data from Table 1. Also assume all onboard electrical systems are turned off. (3 p)

	Absorption coefficient	Emission coefficient
Solar panels	0.80	0.90
Thermal blanket	0.20	0.90

Table 1: Some thermal material properties.

Solution:

The spacecraft temperature will be determined by the balance between absorbed sunlight radiation power P_a and emitted thermal radiation power P_e. The absorbed power is

$$P_\mathrm{a} = A_\mathrm{a}\alpha I_0 \quad (1)$$

where $I_0 = 1.4\ \mathrm{kW/m^2}$ is the solar radiation intensity, α is the absorbtion coefficient, and A_a is the absorbtion area. We can note that only the solar panels are exposed to the sunlight, as the top and bottom sides do not see any sunlight due to the satellite symmetry axis being perpendicular to the solar direction. From the sun, the satellites looks like rectangles with height $h = 1$ m and sides $2r = 3$ m, all of which are covered with solar panels, so that

$$A_{\rm a} = 2rh; \tag{2}$$

thus

$$P_{\rm a} = 2rh\alpha_{\rm solarpanel} I_0. \tag{3}$$

From the table, we see that all surfaces have the same emission coefficient $\epsilon = 0.90$. Nothing else is given, so we thus assume that the total emission area is the sum of the mantle, top and bottom areas, i.e.

$$A_{\rm e} = 2\pi rh + \pi r^2 + \pi r^2 = 2\pi r(r+h), \tag{4}$$

so that

$$P_{\rm e} = A_{\rm e}\epsilon\sigma T^4 = 2\pi r(r+h)\epsilon\sigma T^4. \tag{5}$$

Balancing absorbed and emitted power, we get

$$T = \left(\frac{I_0}{\sigma}\frac{2rh}{2\pi r(r+h)}\frac{\alpha_{\rm solarpanel}}{\epsilon}\right)^{1/4} = \left(\frac{I_0}{\sigma}\frac{h}{\pi(r+h)}\frac{\alpha_{\rm solarpanel}}{\epsilon}\right)^{1/4}, \tag{6}$$

which on insertion of numerical values yields

$$T = \left(\frac{1.4\cdot 10^3}{5.67\cdot 10^{-8}}\cdot\frac{1}{\pi(1.5+1)}\cdot\frac{0.8}{0.9}\right)^{1/4} \text{ K} = 230 \text{ K} = -43°\text{C}. \tag{7}$$

Note: In reality the thermal blankets stops most of the emission, so that the effective emissive area is $2\pi rh$: the equilibrium temperature thus becomes 16°C, which is further increased by all electrical devices onboard.

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5. (a) When launching a spacecraft into circular orbit at altitude h above the ground, the rocket must do work to increase the gravitational potential energy by some amount ΔU as well as to acquire the kinetic energy K corresponding to this orbit. Derive an expression for the ratio $\Delta U/K$ as a function of h. In what circumstances can one neglect either ΔU or K? (3 p)

   (b) Providing a spacecraft with a magnetosphere of its own, by using an on-board electromagnet (possibly superconducting), is a suggested method for solar wind sailing. Estimate the dipole moment (in units of A·m^2) needed to provide a force balancing the solar gravitation at Earth orbit for a 20 kg microsatellite. Typical parameters for the solar wind can be taken to be $n_{\rm e} = 5$ cm^{-3} and $v = 400$ km/s. (3 p)

(c) Is the result in (b) important in practice? Is it necessary that $F_{\rm sail} > F_{\rm g}$ if one wants to use this technique to travel outward through the solar system? Explain your answer. (1 p)

Solution:

(a) The gravitational potential energy of a spacecraft of mass m orbiting a planet of mass $m_{\rm E}$ at distance r is

$$U = -G\frac{m_{\rm E}m}{r} + \text{constant}, \tag{1}$$

so the extra potential energy increase is

$$\Delta U = Gm_{\rm E}m\left(\frac{1}{R_{\rm E}} - \frac{1}{r}\right). \tag{2}$$

The orbital kinetic energy is obtained from the requirement that the necessary centripetal force for the circular orbit should be provided by the gravitational force,

$$\frac{mv^2}{r} = G\frac{m_{\rm E}m}{r^2}, \tag{3}$$

so that

$$K = \frac{1}{2}mv^2 = G\frac{m_{\rm E}m}{r^2}. \tag{4}$$

The desired ratio thus is

$$\frac{\Delta U}{K} = \frac{\frac{Gm_{\rm E}m}{R_{\rm E}} - \frac{Gm_{\rm E}m}{r}}{\frac{Gm_{\rm E}m}{2r}} = 2\frac{r}{R_{\rm E}} - 2 = 2h/R_{\rm E} \tag{5}$$

where we have used that $r = R_{\rm E} + h$. Obviously, one may neglect ΔU for low altitude orbits ($h \ll R_{\rm E}$), while K can be neglected in the opposite case $(h \gg R_{\rm E})$.

(b) We are only deriving an estimate, not an exact expression, so we may assume $\theta = 90°$ and estimate the field strength in the dipole field by

$$B(r) = \frac{\mu_0 M}{4\pi r^3}. \tag{6}$$

This gives us a magnetic pressure

$$p_{\rm B} = \frac{1}{2\mu_0}B^2 = \frac{\mu_0 M^2}{32\pi^2 r^6} \tag{7}$$

to balance the solar wind momentum flux

$$p_{\rm v} = m_{\rm ion} n v^2, \tag{8}$$

where $m_{\text{ion}} = 0.75\, m_{\text{H}} + 0.25\, m_{\text{He}} \approx 1.75\, m_{\text{p}}$ is the typical ion mass in the solar wind, and n and v are defined in the problem text. We thus get a typical size

$$r = \left(\frac{\mu_0 M^2}{32\pi^2 mnv^2} \right)^{1/6} \tag{9}$$

of our artificial magnetosphere. The force provided by the solar wind should then be on the order of

$$F_{\text{sail}} \sim \pi p_{\text{v}} r^2 = \pi n m_{\text{ion}} v^2 \left(\frac{\mu_0 M^2}{32\pi^2 m_{\text{ion}} nv^2} \right)^{1/3} = \left(\frac{\pi \mu_0 M^2 m_{\text{ion}}^2 n^2 v^4}{32} \right)^{1/3} . \tag{10}$$

The solar gravitation force is

$$F_{\text{g}} = G \frac{m m_{\text{sun}}}{R^2} \tag{11}$$

where $m_{\text{sun}} = 2 \cdot 10^{30}$ kg is the solar mass and $R = 150 \cdot 10^6$ km the mean distance to the sun, so balancing the forces finally gives

$$M \sim \frac{4}{m_{\text{ion}} nv^2 R^3} \sqrt{\frac{2G^3 m_{\text{sun}}^3 m^3}{\pi \mu_0}} \tag{12}$$

which at input of numbers provides us with the value

$$M \sim 10^{12}\ \text{Am}^2 \tag{13}$$

(which is an enormous value, corresponding to a current of 1 GA flowing through a 1000-loop-coil of area 1 m^2).

(c) This result is not very important, as a spacecraft launched from the Earth automatically gets the Earth's orbital speed around the sun, and thus starts from a close-to-circular orbit. Any small outward force, acting under a long time, can be used for acquiring speed towards the outer parts of the solar system. It is not necessary to have $F_{\text{sail}} > F_{\text{g}}$, so the huge value predicted in (b) does not mean that the method is unfeasible (in reality, one is also considering to further inflate the artificial magnetosphere by using a plasma gun).