Space Physics Formulas:
Complement to Physics Handbook

Charge density and current density from particle species s:

$$\rho = \sum_s q_s n_s, \qquad \mathbf{j} = \sum_s q_s n_s \mathbf{v_s}$$

Galilean transformations:

$$\mathbf{E}' = \mathbf{E} + \mathbf{v} \times \mathbf{B}, \qquad \mathbf{B}' = \mathbf{B}$$

Dipole magnetic field:

$$\mathbf{B}(r,\theta) = -B_0 \left(\frac{R_0}{r}\right)^3 \left(2\hat{\mathbf{r}}\cos\theta + \hat{\boldsymbol{\theta}}\sin\theta\right)$$

Dipole field lines:

$$r/\sin^2\theta = \mathrm{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particle species s:

$$m_s n_s \frac{d\mathbf{v_s}}{dt} = n_s q_s(\mathbf{E} + \mathbf{v_s} \times \mathbf{B}) - \nabla p_s + \text{o.f.}$$

MHD equation of motion:

$$\rho_\mathrm{m}\frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{o.f.} = -\nabla\left(p + \frac{B^2}{2\mu_0}\right) + \frac{1}{\mu_0}(\mathbf{B}\cdot\nabla)\,\mathbf{B} + \text{o.f.}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Dynamic pressure:

$$p_\mathrm{dyn} = \frac{1}{2}nmv^2$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_\mathrm{P} & -\sigma_\mathrm{H} & 0 \\ \sigma_\mathrm{H} & \sigma_\mathrm{P} & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_\parallel \end{pmatrix} = \sigma_\mathrm{P}\mathbf{E}_\perp + \sigma_\mathrm{H}\frac{\mathbf{B} \times \mathbf{E}_\perp}{B} + \sigma_\parallel \mathbf{E}_\parallel$$

Conductivities:

$$\begin{aligned} \sigma_\mathrm{P} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}\nu_\mathrm{i}}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} + \frac{\omega_\mathrm{ce}\nu_\mathrm{e}}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\mathrm{H} &= \frac{ne}{B}\left(\frac{\omega_\mathrm{ci}^2}{\omega_\mathrm{ci}^2+\nu_\mathrm{i}^2} - \frac{\omega_\mathrm{ce}^2}{\omega_\mathrm{ce}^2+\nu_\mathrm{e}^2}\right) \\ \sigma_\parallel &= ne^2\left(\frac{1}{m_\mathrm{i}\nu_\mathrm{i}} + \frac{1}{m_\mathrm{e}\nu_\mathrm{e}}\right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_{\mathrm{c}} = \omega_{\mathrm{c}}/(2\pi) = \frac{1}{2\pi}\frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2}mv_{\perp}^{2}/B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_{B} = -\mu\nabla B$$

Drift motion due to general force $\mathbf{F}$:

$$\mathbf{v}_{\mathbf{F}} = \frac{\mathbf{F}\times\mathbf{B}}{qB^{2}}$$

Pitch angle:

$$\tan\alpha = v_{\perp}/v_{\parallel}$$

Electrostatic potential from charge Q in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_{0}}\frac{e^{-r/\lambda_{\mathrm{D}}}}{r}$$

Debye length:

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\epsilon_{0}KT}{ne^{2}}}$$

Plasma frequency:

$$f_{\mathrm{p}} = \omega_{\mathrm{p}}/(2\pi) = \frac{1}{2\pi}\sqrt{\frac{ne^{2}}{\epsilon_{0}m_{\mathrm{e}}}}$$

Rocket thrust:

$$T = v_{\mathrm{e}}\frac{\mathrm{d}m}{\mathrm{d}t}$$

Specific impulse:

$$I_{\mathrm{sp}} = \frac{\int T\,\mathrm{d}t}{m_{\mathrm{fuel}}g} = v_{\mathrm{e}}/g$$

The rocket equation:

$$\Delta v = -gt_{\mathrm{burn}} + v_{\mathrm{e}}\ln\left(1 + \frac{m_{\mathrm{fuel}}}{m_{\mathrm{payload+structure}}}\right)$$

Total energy of elliptic orbit of semimajor axis a:

$$E = -\frac{GMm}{2a}$$

Emitted thermal radiation power:

$$P_{\mathrm{e}} = \varepsilon\sigma A_{\mathrm{e}}T^{4}$$

Absorbed solar radiation power:

$$P_{\mathrm{a}} = \alpha A_{\mathrm{a}}I_{\mathrm{rad}}$$

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