

# Space Physics Formulas: Complement to Physics Handbook

Charge density in plasma with charge particle species  $s$ :

$$\rho = \sum_s q_s n_s$$

Current density:

$$\mathbf{j} = \sum_s q_s n_s \mathbf{v}_s$$

Dipole magnetic field:

$$\mathbf{B}(r, \theta) = -B_0 \left( \frac{R_0}{r} \right)^3 \left( 2\hat{\mathbf{r}} \cos \theta + \hat{\theta} \sin \theta \right)$$

Dipole field lines:

$$r / \sin^2 \theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_m \frac{d\mathbf{v}}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particles:

$$mn \frac{d\mathbf{v}}{dt} = nq(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \nabla p + \text{other forces}$$

MHD equation of motion:

$$\rho_m \frac{d\mathbf{v}}{dt} = \mathbf{j} \times \mathbf{B} - \nabla p + \text{other forces}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Condition for "frozen-in" magnetic field:

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Ohm's law:

$$\mathbf{j} = \begin{pmatrix} \sigma_P & \sigma_H & 0 \\ -\sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_{\parallel} \end{pmatrix} \begin{pmatrix} E_{\perp} \\ 0 \\ E_{\parallel} \end{pmatrix}$$

Conductivities:

$$\begin{aligned} \sigma_P &= \frac{ne}{B} \left( \frac{\omega_{ci}\nu_i}{\omega_{ci}^2 + \nu_i^2} + \frac{\omega_{ce}\nu_e}{\omega_{ce}^2 + \nu_e^2} \right) \\ \sigma_H &= \frac{ne}{B} \left( \frac{\omega_{ci}^2}{\omega_{ci}^2 + \nu_i^2} - \frac{\omega_{ce}^2}{\omega_{ce}^2 + \nu_e^2} \right) \\ \sigma_{\parallel} &= ne^2 \left( \frac{1}{m_i\nu_i} + \frac{1}{m_e\nu_e} \right) \end{aligned}$$

Cyclotron frequency (gyrofrequency):

$$f_c = \omega_c / (2\pi) = \frac{1}{2\pi} \frac{qB}{m}$$

Magnetic moment of charged particle gyrating in magnetic field:

$$\mu = \frac{1}{2} m v_{\perp}^2 / B$$

Magnetic force on magnetic dipole:

$$\mathbf{F}_B = -\mu \nabla B$$

Drift motion due to general force  $\mathbf{F}$ :

$$\mathbf{v}_F = \frac{\mathbf{F} \times \mathbf{B}}{qB^2}$$

Pitch angle:

$$\tan \alpha = v_{\perp}/v_{\parallel}$$

Electrostatic potential from charge  $Q$  in a plasma:

$$\Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_D}}{r}$$

Debye length:

$$\lambda_D = \sqrt{\frac{\epsilon_0 K T}{ne^2}}$$

Plasma frequency:

$$f_p = \omega_p/(2\pi) = \frac{1}{2\pi} \sqrt{\frac{ne^2}{\epsilon_0 m_e}}$$

Rocket thrust:

$$T = v_e \frac{dm}{dt}$$

Specific impulse:

$$I_{sp} = \frac{\int T dt}{m_{fuel} g} = v_e/g$$

The rocket equation:

$$\Delta v = -gt_{burn} + v_e \ln \left( 1 + \frac{m_{fuel}}{m_{payload+structure}} \right)$$

Emitted thermal radiation power:

$$P_e = \epsilon \sigma A_e T^4$$

Absorbed solar radiation power:

$$P_a = \alpha A_a I_{rad}$$