Space Physics Formulas: Complement to Physics Handbook

Charge density and current density from particle species $s$:

$$\rho = \sum_s q_s n_s,$$
$$j = \sum_s q_s n_s v_s$$

Galilean transformations:

$$E' = E + v \times B, \quad B' = B$$

Dipole magnetic field:

$$B(r, \theta) = -B_0 \left( \frac{R_0}{r} \right)^3 \left( 2\hat{r}\cos \theta + \hat{\theta} \sin \theta \right)$$

Dipole field lines:

$$r / \sin^2 \theta = \text{const.}$$

Magnetic field energy density and pressure:

$$w_B = p_B = \frac{B^2}{2\mu_0}$$

Equation of motion of neutral gas:

$$\rho_m \frac{dv}{dt} = -\nabla p + \text{other forces}$$

Equation of motion of gas of charged particle species $s$:

$$m_s n_s \frac{dv_s}{dt} = n_s q_s (E + v_s \times B) - \nabla p_s + \text{o.f.}$$

MHD equation of motion:

$$\rho_m \frac{dv}{dt} = j \times B - \nabla p + \text{o.f.} \Rightarrow -\nabla \left( p + \frac{B^2}{2\mu_0} \right) + \frac{1}{\mu_0} (B \cdot \nabla) B + \text{o.f.}$$

Equation of continuity:

$$\frac{\partial n}{\partial t} + \nabla \cdot (n v) = Q - L$$

Equation of state for ideal gas:

$$p = nKT$$

Dynamic pressure:

$$p_{\text{dyn}} = \frac{1}{2} n m v^2$$

Condition for “frozen-in” magnetic field:

$$E + v \times B = 0$$

Ohm’s law:

$$j = \begin{pmatrix} \sigma_P & -\sigma_H & 0 \\ \sigma_H & \sigma_P & 0 \\ 0 & 0 & \sigma_\parallel \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_\parallel \end{pmatrix} = \sigma_P E_\perp + \sigma_H \frac{B \times E_\perp}{B} + \sigma_\parallel \frac{E_\parallel}{B}$$

Conductivities:

$$\sigma_P = \frac{ne}{B} \left( \frac{\omega_{ce} v_e}{\omega_{ce}^2 + v_e^2} + \frac{\omega_{ci} v_i}{\omega_{ci}^2 + v_i^2} \right)$$
$$\sigma_H = \frac{ne}{B} \left( \frac{\omega_{ce}^2}{\omega_{ce}^2 + v_e^2} - \frac{\omega_{ci}^2}{\omega_{ci}^2 + v_i^2} \right)$$
$$\sigma_\parallel = ne^2 \left( \frac{1}{m_e v_e} + \frac{1}{m_i v_i} \right)$$
Cyclotron frequency (gyrofrequency):
\[ f_c = \frac{\omega_c}{(2\pi)} = \frac{1}{2\pi} \frac{qB}{m} \]

Magnetic moment of charged particle gyrating in magnetic field:
\[ \mu = \frac{1}{2} mv_{\perp}^2 / B \]

Magnetic force on magnetic dipole:
\[ \mathbf{F}_B = -\mu \nabla B \]

Drift motion due to general force \( \mathbf{F} \):
\[ \mathbf{v}_F = \frac{\mathbf{F} \times \mathbf{B}}{qB^2} \]

Pitch angle:
\[ \tan \alpha = \frac{v_{\perp}}{v_{\parallel}} \]

Electrostatic potential from charge \( Q \) in a plasma:
\[ \Phi(r) = \frac{Q}{4\pi\epsilon_0} \frac{e^{-r/\lambda_D}}{r} \]

Debye length:
\[ \lambda_D = \sqrt{\frac{\epsilon_0 KT}{ne^2}} \]

Plasma frequency:
\[ f_p = \frac{\omega_p}{(2\pi)} = \frac{1}{2\pi} \sqrt{\frac{ne^2}{\epsilon_0 m_e}} \]

Rocket thrust:
\[ T = v_e \frac{dm}{dt} \]

Specific impulse:
\[ I_{sp} = \int \frac{T dt}{m_{fuel} g} = v_e / g \]

The rocket equation:
\[ \Delta v = -gt_{\text{burn}} + v_e \ln \left( 1 + \frac{m_{\text{fuel}}}{m_{\text{payload+structure}}} \right) \]

Total energy of elliptic orbit of semimajor axis \( a \):
\[ E = -\frac{GMm}{2a} \]

Emitted thermal radiation power:
\[ P_c = \varepsilon \sigma A_e T^4 \]

Absorbed solar radiation power:
\[ P_a = \alpha A_s I_{\text{rad}} \]