Exam in *Antenna Theory*
Time: 19 March 2007 at 09.00–14.00.
Location: Gimogatan 4, Sal 1

You may bring: Laboratory reports, pocket calculator, English dictionary,
Råde-Westergren: "Beta", Nordling-Österman: "Physics Handbook", or comparable handbooks.

Six problems, maximum five points each, for a total maximum of 30 points.

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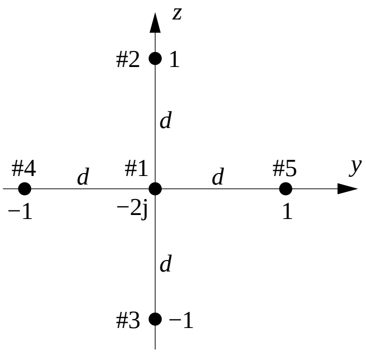
1. Two antennas are separated by a distance R. Derive the equation which relates the power received to the power transmitted between these two antennas (Friis transmission equation).

2. Consider a very thin dipole of finite length l, positioned symmetrically about the origin and with its length directed along the z-axis. Show that in order to maintain a maximum phase error of an antenna equal to or less than $\left(\frac{\pi}{8}\right)$, the observation distance must be equal to or greater than $\left(\frac{2l^2}{\lambda}\right)$ (the so called far-field approximation).
   Hint: The vector potential $\mathbf{A}$ is given by
   $$\mathbf{A} = \frac{\mu}{4\pi} \int \mathbf{I}_\mathrm{e}(x', y', z') \frac{\mathrm{e}^{-\mathrm{j}kR}}{R} \mathrm{d}l'$$

3. An array of isotropic sources consists of five elements, spaced according to the Figure below with equal spacing d between the elements. The elements have excitations (phases and amplitudes) as shown in the Figure.

   a) Find the array factor.
   b) Find the smallest spacing d such that a null is formed in the $\phi = 0°$ plane at an angle of $\theta = 45°$ from the z axis.



4. An infinitesimal horizontal magnetic dipole is placed parallel to the y axis a height $h = \lambda/2$ above an infinite electric ground plane.

   a) Find the far-zone **E**- and **H**-field components radiated by the dipole in the presence of the ground plane.
   b) For the elevation plane $\phi = 30°$, find all the nulls of the total field.

   Hint: The far-field components radiated from an infinitesimal electric dipole, of length l, placed symmetrically about the origin and directed along the z axis are given by
   $$\mathbf{E} = \hat{\boldsymbol{\theta}} E_\theta, \quad \text{where} \quad E_\theta = \mathrm{j}\eta \frac{kI_0 l}{4\pi r} \mathrm{e}^{-\mathrm{j}kr} \sin\theta,$$
   $$\mathbf{H} = \hat{\boldsymbol{\phi}} H_\phi, \quad \text{where} \quad H_\phi = E_\theta/\eta.$$

Continued. P.t.o.→

5. Design a constant current circular loop so that its far-field pattern has a null in the plane of the loop, and three nulls above and three nulls below the plane of the loop.

   a) Find the radius of the loop.
   b) Find the angles where the nulls occur.

   Hint: The far-field components radiated by a circular loop of radius a and constant current I_0 are given by

$$
\begin{aligned}
E_\phi &= \frac{a\omega\mu I_0}{2r}\mathrm{e}^{-\mathrm{j}kr}J_1(ka\sin\theta), \\
H_\theta &= -E_\phi/\eta,
\end{aligned}
$$

   where $J_1(z)$ is the Bessel function of the first order. Nulls of the Bessel function $[J_1(z_n) = 0]$ occur at z_1=0, z_2=3.83, z_3=7.02, z_4=10.17, . . .

6. Design a linear array of isotropic elements placed along the z axis such that the nulls of the array factor occur at $\theta = 60°$, $\theta = 90°$, and $\theta = 120°$. Assume that the elements are spaced a distance $d = \lambda/2$ apart and that $\beta = \pi/2$.

   a) Sketch and label the visible region on the unit circle.
   b) Find the required number of elements.
   c) Determine the excitation coefficients of the elements.

   Hint: The array factor of an N-element linear array is given by

$$
AF = \sum_{n=1}^{N} a_n e^{\mathrm{j}(n-1)\psi},
$$

   where $\psi = kd\cos\theta + \beta$. Use the representation $z = e^{\mathrm{j}\psi}$.
