Exam in *Antenna Theory*
Time: 17 March 2009, at 8.00–13.00.
Location: Gimogatan 4, Sal 2

You may bring: Laboratory reports, pocket calculator, English dictionary, Råde-Westergren: "Beta", Nordling-Österman: "Physics Handbook", or comparable handbooks.

Six problems, maximum five points each, for a total maximum of 30 points.

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1. Consider an infinitesimal electric dipole of length l placed symmetically at the origin. Assume that the current in the dipole is constant and given by $\mathbf{I}_\mathrm{e}(z') = \hat{\mathbf{z}} I_0$. Find the electric and magnetic field components radiated by the dipole in *all* space.
   Hint: The vector potential $\mathbf{A}$ is given by

   $$\mathbf{A} = \frac{\mu}{4\pi} \int \mathbf{I}_\mathrm{e}(x', y', z') \frac{e^{-\mathrm{j}kR}}{R} \mathrm{d}l'$$

   where R is the distance from any point on the source to the observation point.

2. The normalized far-zone field pattern of an antenna is given by

   $$E = \begin{cases} \sqrt{\sin\theta \sin^2\phi} & 0 \le \theta \le \pi, \ \ 0 \le \phi \le \pi \\ 0 & \text{elsewhere} \end{cases}$$

   Find the directivity using

   a) the exact expression;
   b) the most appropriate approximative formula.
   c) Explain why your chosen approximative formula gives the best value. When should the other approximative formula be prefered?

3. A vertical infinitesimal linear electric dipole of length l is placed a distance h above an infinite perfectly conducting electric ground plane.

   a) Find the electric and magnetic fields radiated by the dipole in the presence of the ground plane.
   b) Determine the height h above the conducting plane at which the dipole must be elevated so that nulls are formed at angles $\theta = 0^\circ$ and $\theta = 60^\circ$ from the direction normal to the conducting plane.

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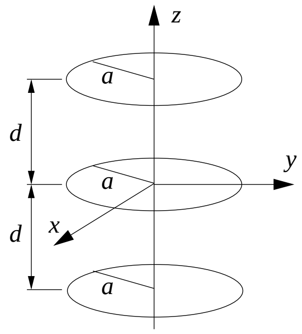
4. Three constant current circular loops of equal radii $a = 7\lambda/10$ and equal current amplitudes and phases are placed along the z axis with the planes ot the loops in the xy plane, see figure below. The spacing between the loops is uniform and equal to $d = \lambda/2$. If the loops are assumed not to couple to each other, find the nulls of the far-field pattern radiated by the loops.



5. Design a linear array of isotropic elements placed along the z axis such that the nulls of the array factor occur at $\theta = 0^\circ$, 60°, 90°, 120°, and 180°. Assume that the elements are spaced a distance $d = \lambda/2$ apart and that $\beta = 0$.

   a) Sketch and label the visible region on the unit circle.
   b) Find the smallest possible number of required elements and their excitation coefficients.
   c) Determine the length of the array.

   Hint: The array factor of an N-element linear array is given by $AF = \sum_{n=1}^{N} a_n e^{j(n-1)\psi}$, where $\psi = kd\cos\theta + \beta$. Use the representation $z = e^{j\psi}$.

6. A rectangular aperture of dimensions a and b is placed at an infinite ground plane as shown in the figure below. The tangential field distribution over the aperture is given by

$$\mathbf{E}_a = \hat{x}E_0 \qquad \begin{array}{l} -b/2 \le x' \le b/2 \\ -a/2 \le z' \le a/2 \end{array}$$

   Find the spherical far-zone electric and magnetic field components radiated by the aperture. The spherical field components must be expressed with respect to the coordinate system specified in the figure.

   Hint:

$$\int_{-c/2}^{c/2} e^{j\alpha x} \mathrm{d}x = c\,\frac{\sin\left(\frac{\alpha}{2}c\right)}{\frac{\alpha}{2}c}$$